Amendments, Covering, and Agenda Control: The Politics of Open Rules*

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Abstract

Agenda control in legislative bodies is an important and complicated topic. Frequently, the discussion of agenda control is limited to procedures that restrict what may be considered on the floor, as exemplified by "closed rules" in the United States House of Representatives. In this paper, we expand the discussion of agenda control by considering the effects of predetermining part of an amendment agenda. Empirically, this type of agenda control is practiced through the use of an "open" rule that provides a privileged position on the voting agenda to one or more amendments. By defining a binary relation for general collective choice situations, the *needing relation*, we characterize the feasibility of agenda control under open rules with sophisticated legislators. We demonstrate an equivalence between a notion of majoritarianism and outcomes that do not depend upon agenda manipulation to be selected under an open rule. Specifically, we demonstrate that this set of outcomes is the "Banks set of the Banks set."

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As described famously and colorfully by Rep. John Dingell (D-MI),[1] the structure of collective choice process is at least as important as the content of the alternatives from which the choice is made and, furthermore, legislators know this. Indeed, this reality is the basis of the power of the Committee on Rules in the U.S. House of Representatives since the 1880s.[2] Similarly, and with apologies to Prof. Lasswell, the ubiquitous importance of procedure in the determination of policy outcomes motivates the substantial scholarly literature considering the "who, what, when, and how" of special rules within the modern House.[3] The majority of attention in this literature, however, is focused on restrictive or "closed" rules: those that (in *de jure* terms) limit the amendments that may be offered on the floor. This focus is not surprising, given the conundrum presented the frequency with which such rules receive (majority) approval. However, the relative lack of attention paid to open rules – one in which any number of (germane) amendments are allowed – may suggest that these rules are universally and uniformly concordant with the expression of majority will on the floor. But not all open rules are created equal: many open rules prespecify the order in which certain amendments will be considered.[4] Since such rules (at least partially) order the consideration of amendments, we refer to rules that do not restrict floor amendments but do prespecify one or more amendments as *ordered open rules*.

While it is well known that collective choice is sensitive to the choice of procedures, the ways by which an institutional innovator can manipulate collective choice

[1] "I'll let you write the substance. . .and you let me write the procedure, and I'll screw you every time." *C.f.*, Regulatory Reform Act: Hearings on H.R. 2327 Before the Subcommittee on Administrative Law and Governmental Relations of the House Committee on the Judiciary, 98th Cong. 312 (1983).

[2] See, for example, Sullivan [1984], Bach and Smith [1988], and Schickler [2002].

[3] The literature discussing special rules is enormous and constantly growing. Some important contributions include Shepsle and Weingast [1984b], Rohde [1991], Dion and Huber [1996, 1997], Dion [1997], Evans [1999], Marshall [2002], Roberts and Smith [2003], and Roberts [2005], among others.

[4] Consider, for example, the categorizations employed by Bach and Smith [1988], Krehbiel [1991], Sinclair [1999], Marshall [2002], and Oleszek [2007], as well as by the House Rules Committee (*inter alia*, U.S. House of Representatives [1996]).

are frequently not clear. For example, the forced inclusion of more alternatives on the agenda can reduce the set of outcomes that may be achieved through the proposal of amendments and sophisticated voting. Furthermore, alternatives that are themselves not feasible as collective choices may nonetheless play a role in collective choice. This fact highlights once again the stark contrast between collective and individual choice (*e.g.*, Plott [1973]). Similarly, we show below that this creates the potential for a strategic agenda setter to require that a set of amendments to be voted on by the floor as amendments to the bill *per se*, rather than simply inserting these amendments into the bill itself through the use of a "self-executing" rule.[5] We show that this type of agenda control is present *even when the legislators are assumed to vote sophisticatedly*.

Viewed more broadly, the conclusion of the paper is that the observation of "more" opportunities for apparently majoritarian choice may actually indicate an arguably counter-majoritarian process at work. Relatedly, we show that – even when there is no Condorcet winner (*i.e.*, the majority rule core is empty) – the Rules Committee can report an (ordered) open rule that guarantees the selection of any alternative in the Banks set (Banks [1985]). Furthermore, we demonstrate that the use of an ordered open rule will be necessary to do this in the sense that a self-executing rule automatically incorporating the desired outcome into the bill will lead to the selection of a different alternative on the floor. Accordingly, *our theory implies that the degree of agenda control being exercised by the Rules Committee with respect to any given bill is not solely a function of the "restrictiveness" of the rule reported by the Committee.* To provide motivation for our theory, we now discuss a recent (and typical) example of the use of an ordered open rule in the U.S. House of Representatives.

[5] See Oleszek [2006] for more on self-executing rules and recent examples of their use.

1 Modified Open Rules and Substitute Bills

In February of 2007, the House of Representatives considered H.R. 556 (the "National Security Foreign Investment Reform and Strengthened Transparency Act") under the provisions of H.Res. 195, a modified open rule. In many ways, H.Res. 195 is similar to most non-closed special rules adopted so far during the 110th Congress. It allows amendments that are preprinted in the *Congressional Record* in accord with House Rule XVIII and, most importantly for the purposes of this paper, provides for consideration of a committee-sponsored amendment in the nature of a substitute (henceforth, an ANS).[6] The requirement for preprinted amendments is commonplace and essentially reduces the uncertainty faced by the members of the Rules Committee when designing the rule (Bach and Smith [1988]). Less widely discussed is the Rules Committee's choice to provide for consideration of the committee's amendment as a substitute, rather than providing for its immediate adoption, as was done in H.Res. 254 (which provided for the consideration of H.R. 1227). The relevant portion of H.Res. 254 reads as follows:

> "The amendment in the nature of a substitute recommended by the Committee on Financial Services now printed in the bill, modified by the amendment printed in part A of the report of the Committee on Rules accompanying this resolution, shall be considered as adopted in the House and in the Committee of the Whole. The bill, as amended, shall be considered as the original bill for the purpose of further amendment under the five-minute rule and shall be considered as read." [H.Res. 254, 110th Congress, 1st Session]

[6]The amendment was recommended by the Committee on Financial Services, the committee of primary referral for H.R. 556.

Six amendments, two proposed by Democrats and four proposed by Republicans, were offered to the committee's ANS.[7] In order to simplify the discussion, however, we will represent the situation as consisting of four possible alternatives: the status quo (denoted by Q), the original bill (B), the committee's ANS (C), and the floor-amended version of the committee's ANS (F). Now suppose that the majority preference relation (denoted by $\succ$) in the 104th House over these four alternatives, was as follows:

$$\begin{aligned} &F \succ C \succ B \succ F, \text{ (a cycle)} \\ &F \succ Q, \quad C \succ Q, \quad \text{and} \quad B \succ Q. \end{aligned} \tag{1}$$

This majority preference relation is displayed in Figure 1, where the notation $F \rightarrow Q$ represents $F \succ Q$.

Why Bother With the Original Bill At All? To understand the strategic incentives of an agenda-setter (*i.e.*, the Rules Committee) in this example, denote an amendment agenda consisting of x being pitted against y, the winner of which is pitted against z by (x, y, z), an agenda in which x is pitted against y, the winner of this against z, and the winner of that against w by (x, y, z, w), and so forth. Thus, in the case of H.R. 556, the observed agenda was (F, C, B, Q): the status quo is always the last item voted on in an amendment agenda, the committee-supported ANS was proposed as the first amendment, and the floor amendments were proposed as the final change to the ANS. Following the well-known algorithm of Shepsle and Weingast [1984a], note that the sophisticated voting outcome of (F, C, B, Q) is C. Now, consider for

[7] The two Democrat-sponsored amendments were a manager's amendment offered by Rep. Frank (D-MA) and an amendment offered by Rep. Barrow (D-MA). Each of these was approved by a voice vote. Among the four GOP-sponsored amendments were three amendments proposed by Rep. McCaul (R-TX) calling for reports on the burdens imposed on businesses by government regulations and taxes (all of which failed) and an amendment proposed by Rep. King (R-IA) requiring the President to "consider the potential effects of a a covered transaction on the efforts of the United States to curtail human smuggling," which was approved by a voice vote.

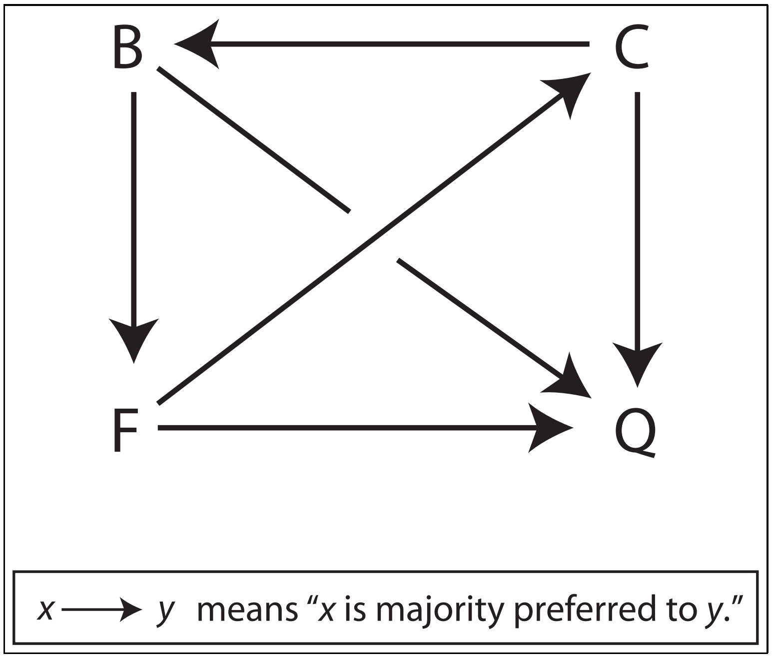

Figure 1: A Cyclic Majority Preference

a moment – given the hypothesized majority preference relation $\succ$ – why the Rules Committee did not provide a modified open "self-executing" rule in which the text of C was automatically substituted for B upon approval of the rule and then allowed for amendments to C as if it were the original bill itself. In this case, the final voting agenda would have been (F, C, Q), which yields a sophisticated voting outcome of F. Furthermore, note that the voting order itself is important: the sophisticated voting outcome of (C, F, B, Q) is also F. Accordingly, C must be the first amendment to B in order to be the sophisticated voting outcome under an open rule.

Intuitively, and following the language of the theory presented below, the committee's amendment, C, "needs" the original bill, B, so long as F is allowed on the voting agenda (*i.e.*, as long as an open rule is used for the floor consideration of B). Of course, the Committee on Financial Services could have written a new bill identical to C, introduced it and had it referred back to itself.[8] Then the committee could have sought

[8]Under the Rules of the modern House, the Committee on Financial Services does not have (and has

a closed rule for consideration of C on the floor in order to keep F off the voting agenda. (Similarly, the Rules Committee could have written a self-executing closed rule in which B would be automatically replaced by C and C then pitted against Q with no possibility for amendment.) One plausible explanation for why neither of these occurred is that securing approval of a closed rule (especially for consideration of a fairly high visibility bill) is more difficult than securing approval of an open rule.[9] Notice that any additional cost of a closed rule would lead the Rules Committee to prefer an open rule with the committee's amendment being treated as an ANS, insofar as treating the amendment in this way results in the same outcome as a closed rule. In other words, in this example, an instrumentally-minded Rules Committee had nothing to gain from reporting a rule that was more restrictive than H.Res. 195.

In the remainder of the paper, we generalize the intuitions of the example described above for collective choice over arbitrary finite sets of alternatives. The generalization describes (in terms of the majority preference relation) the types of alternatives that are "like" C and B in the example above. Specifically, the theory developed in this paper highlights a special dependence of some alternatives on others in terms of being selected through sophisticated voting over an amendment agenda. Described informally, the theory illustrates the strategic importance of how sensitive an alternative is to the exact choice of the voting agenda. The study of agendas is intimately tied to the study of covering, to which we turn in the next section.

not had) the authority to initiate legislation, *per se*.

[9] Among many others, the works of Krehbiel, Shepsle, and Weingast [1987], Bach and Smith [1988], Krehbiel [1991], Cox and McCubbins [1993, 2005], Sinclair [1995], Binder [1997], Schickler [2002], Crombez, Groseclose, and Krehbiel [2006], and Oleszek [2007] discuss the variety of ways in which open rules are less controversial than closed rules, *ceteris paribus*.

2 Covering and Agenda Manipulation

The notion of covering is central to the modern theory of choice and is frequently and justifiably presented as one of the most important limitations on the power of an agenda setter.[10] An alternative x is "covered" by another point y if y defeats x and defeats every alternative that x defeats. An alternative x is *uncovered* if it is not covered by any other alternative y. Miller [1980] demonstrated that uncovered points are precisely those that can beat all other alternatives in at most two steps.[11]

The uncovered set is substantively interesting partly because of its connection with sophisticated voting over binary agendas. In particular, when voting is carried out using an *amendment agenda* with a given status quo policy q, Shepsle and Weingast [1984a] use Miller's "two-step principle" to show that, for any alternative y, there exists an agenda yielding y as the sophisticated voting outcome if and only if q does not cover y.[12] Accordingly, for any status quo policy q and any alternative y in the uncovered set, there exists an amendment agenda $\alpha(q, y)$ such that y is the sophisticated voting outcome of $\alpha(q, y)$ and, furthermore, the uncovered set is exactly the set of alternatives for which this statement is true. Substantively, the uncovered set is the set of alternatives that can always be implemented by some closed rule.

Soon thereafter, Banks [1985] extended the work of Shepsle and Weingast [1984a]

[10] In addition to the seminal developments of Fishburn [1977] and Miller [1980], see also Shepsle and Weingast [1984a], Banks [1985], McKelvey [1986], Feld, Grofman, Hartly, Kilgour, and Miller [1987], Feld and Grofman [1988], Miller, Grofman, and Feld [1990b], Epstein [1997], Schofield [1999], Tsebelis [2002], Bianco, Jeliazkov, and Sened [2004], Dutta, Jackson, and Breton [2004], Bianco and Sened [2005], Bianco, Lynch, Miller, and Sened [2006], Jeong [2007], and Miller [2007].

[11] Miller examines the uncovered set using graph theoretic tools and shows a nesting relationship among various solution concepts. Specifically, Miller shows that the core is a subset of the uncovered set, which is in turn a subset of the top cycle set. Miller assumes that the set of alternatives is finite and that the majority preference relation is strict: both assumptions are maintained in this paper. Penn [2006a,b] relaxes each of these assumptions and shows how the equivalence of several different definitions of the uncovered set disappears when preferences are not strict.

[12] An amendment agenda is a binary voting procedure in which all amendments are voted on in the reverse of the order in which they are proposed, ensuring that the status quo is voted on at the final stage of the voting tree (see Ordeshook and Schwartz [1987] and Rasch [2000]).

to demonstrate that, if one requires that the amendment agenda contain all feasible alternatives, then the set of outcomes that can be chosen as sophisticated winners are a subset (and often a proper subset) of the uncovered subset. This set, now known as the *Banks set*, represents the set of outcomes that can be chosen via an "unamendable" agenda. The Banks set is accordingly an appealing solution concept when the legislature is operating under an open rule. Miller, Grofman, and Feld [1990a] explored the Banks set in more detail, proving several facts about its structure.[13]

The definitions of both the uncovered and Banks sets are based on existence in the following way. For any finite set of alternatives X, an alternative $x \in X$ is uncovered if and only if there exists a subset of the alternatives, Y, with $x \in Y$, such that (1) Y is *externally stable* in X (*i.e.*, for each point $z \notin Y$, there is a point $y \in Y$ such that y is majority preferred to z) and (2) x is the Condorcet winner in Y (*i.e.*, for every point $y \in Y \setminus \{x\}$, x is majority preferred to y). As demonstrated by Banks [1985], an alternative x is in the Banks set if and only if there exists a subset Y with $x \in Y$ such that Y is externally stable, x is the Condorcet winner in Y, *and* Y is a *chain* (*i.e.*, the majority preference restricted to Y is transitive). Manipulating collective choices through the structure of amendment agendas is in some sense a practical (or computational) matter. In other words, simply knowing that an alternative is in either set is not enough to know the set of agendas that one might use to implement it (*i.e.*, *how* to have the alternative selected through sophisticated voting).

[13]Perhaps foremost among the results proved by Miller et al. [1990a] is that the top-cycle set of the Banks set is the Banks set, which plays a subtle role in the link between the Banks set and majoritarian decision making examined later in this paper.

3 Theory

Our understanding of the power of institutions has expanded greatly over the past 30 years but remains incomplete. In particular, despite the dizzying array of voting procedures used in the House of Representatives,[14] the effects of some of the most frequently altered aspects of these procedures are not well understood. As discussed in the introduction, we present a theory of agenda control through the use of ordered open rules. We show that the use of ordered open rules can have dramatic effects on collective choice within bodies such as the U.S. House. Along the way, our results hopefully expand the discussion and exploration of the role and use of special rules – particularly those that are frequently described as "open" – in the U.S. House of Representatives.

To model the situation in as general (and parsimonious) a fashion as possible, we consider a collective choice situation in which an outcome x must be chosen from some finite set of $K \geq 3$ alternatives, X. One of the alternatives is referred to as the status quo, denoted by $q \in X$, and represents the alternative that will be implicitly chosen by the group in the absence of approving some other alternative in X. We denote the majority preference relation by $T \subset X^2$, where $(x, y) \in T$ is written as xTy and indicates that x is majority preferred to y. We assume that T is a *tournament*: a total and asymmetric binary relation on X.[15] In order to keep notation as simple as possible, we will work with the tournament, T, and denote the set of alternatives by X. The set of all tournaments on a set of K alternatives is denoted by $\mathcal{T}_K$ and the set of all finite tournaments is denoted by $\mathcal{T} \equiv \cup_{K=1}^{\infty} \mathcal{T}_K$. For any tournament T

[14] See, among others, Bach and Smith [1988], Ordeshook and Schwartz [1987], Banks [1989], Weingast [1989], and Oleszek [2007].

[15] This assumption rules out ties. As famously shown by McGarvey [1953], any tournament on X can be rationalized as the majority preference relation for a proper selection of individual preferences. It is important to note, however, that McGarvey's theorem relies upon flexibility in the number of individuals. We leave this issue aside for the remainder of the paper.

and alternative $x \in X$, the set of alternatives that defeat x are denoted by $T(x)$ and the set of alternatives that are defeated by x are denoted by $T^{-1}(x)$. For any pair of alternatives $x, y \in X$ and integer $k \geq 1$, we write $xT^k y$ if the length of the shortest path between x and y under T is less than or equal to k. For any tournament T on a set of alternatives X and any integer $k \geq 1$, let $A^k(T)$ denote the k^{th} power of T on X. For any set X and $Y \subset X$, let $X_{-Y} \equiv X \setminus Y$ denote the elements of X not also in Y and, for any tournament T and $Y \subseteq X$, let T_Y denote the subtournament induced by T on Y.[16]

3.1 Minimum Distances & Eccentricity

The graph theoretic treatment of tournaments, initiated by Miller [1977] and extended by many others, allows one to characterize alternatives in terms of how "easily" they can defeat any other alternative under different voting procedures. As a principal example, Ordeshook and Schwartz [1987] consider arbitrary agendas under both sophisticated and sincere voting and demonstrate a key measure of the ease of implementing a given alternative x as the collective choice when y is the status quo is the length of the shortest path from x to y (as defined by T). Accordingly, for any tournament T and any $x, y \in X$, define the following two functions:

$$\begin{aligned} \delta_T(x,y) &= \begin{cases} \min\{k \in \mathbb{Z}_{++} : xT^k y\} & \text{if } \{k \in \mathbb{Z}_{++} : xT^k y\} \neq \emptyset \\ \infty & \text{otherwise.} \end{cases}, \\ e_T(x) &= \max_{y \in X} \delta_T(x,y). \end{aligned}$$

The function $\delta_T(y,x)$ is the minimum distance from y to x (which is defined to be ∞ if no such path exists) and $e_T(y)$ is the (possibly infinite) maximum minimum distance

[16]Formally, for any tournament T and $x, y \in X$, $(x,y) \in T_Y$ if and only if $(x,y) \in Y \cap T$.

for y to defeat every other alternative. The value $e_T(y)$ is referred to as the *eccentricity* of y. The following fact is well-known and implies that the power sets of any tournament T are (weakly) nested: $A^k(T) \subseteq A^{k+1}(T)$ for all $k \geq 1$.

Fact 1 *For any tournament T, $j \geq e_T(x) \Leftrightarrow x \in A^j(T)$.*

3.2 External Stability and Chains

Given a tournament T, a subset $Y \subseteq X$ is

1. *externally stable* if for each $z \in X_{-Y}$, there exists $y_z \in Y$ such that $y_z T z$,
2. a *chain* if T_Y is transitive, and
3. a *maximal chain* if Y is both a chain and externally stable.[17]

Given a tournament T, the set of all externally stable sets under T is denoted by $M(T)$, the set of all chains is denoted by $H(T)$, and the set of all maximal chains is denoted by $MH(T)$. (So, $MH(T) = M(T) \cap H(T)$.) Similarly, for any $x \in X$, the set of all chains with x top-ranked is denoted by $H_T(x)$, the set of all externally stable sets containing x is denoted by $M_T(x)$, and the set of all maximal chains with x top-ranked is denoted by $MH_T(x)$.

3.3 The Top-Cycle, Uncovered, and Banks Sets

The *top-cycle* of a tournament T, denoted by $TC(T)$, is the set of alternatives that can be reached through sincere voting from *any* other alternative in T.[18] As pointed out

[17] Technically, maximality of chains is defined with respect to set inclusion. The two definitions are equivalent in this setting.

[18] This result, which follows from the seminal insights of McKelvey [1976], Miller [1977], and Schofield [1978], is most directly and clearly demonstrated by Ordeshook and Schwartz [1987]. Note also that the top-cycle set is occasionally referred to as the “Condorcet set.” We do not adopt this

by Miller [1980] and others, this set can contain Pareto-dominated alternatives yet, as pointed out by Ordeshook and Schwartz [1987], also identifies the set of alternatives that can be reached – even through sophisticated voting – through many real-world legislative procedures. Formally, $x \in TC(T)$ if and only if $e_T(x) < \infty$.

As mentioned above, the top-cycle set in some sense represents an outer bound for what is achievable through agenda control under legislative procedures. Accordingly, it is not particularly discriminating in terms of generating predictions about what can not occur. As discussed above, more discrimination is accomplished by examining the uncovered set, which we formally define based on the following formal definition of covering.

Definition 1 *For any tournament T and pair of alternatives $x, y \in X$, x* covers *y* with respect to *T, written as xC_Ty, if xTy and $T(x) \cap T(y) = T(x)$ (*i.e., *x beats y and any alternative z that beats x beats y as well).*

The set of alternatives that cover some other alternative $y \in X$ is denoted by $C_T(y)$. The *uncovered set* of T, denoted by $UC(T)$, is then defined as follows:

$$UC(T) = \{x \in X : \{y \in X : yC_Tx\} = \emptyset\},$$

and the set of covered points is denoted by $C(T) \equiv X \setminus UC(T)$.[19] The principle highlights the relationship between covering and outcomes achievable through sophisti-

nomenclature because the justification for its usage – that it is identical to the Condorcet winner when one exists – is also shared by the uncovered set, the Banks set, and the minimal covering set, in addition to all tournament solution concepts (Laslier [1997]).

[19] Note that Miller's "two-step principle" utilized by Definition 1 is an unambiguous basis for the definition of the uncovered set whenever there is a finite number of policies. However, as alluded to earlier in the paper, such an approach may not be a satisfactory definition of the uncovered set in infinite settings, since this approach may not guarantee that the uncovered set is nonempty (Laslier [1997], Penn [2006b]). Nevertheless, since we consider only finite tournaments, this definition is the most convenient and intuitive.

cated voting over amendment agendas, a process central to the theory presented in this paper.

The Banks Set. Extending the work of Shepsle and Weingast [1984a] and others, Banks [1985] demonstrated that sophisticated voting and external stability jointly imply that the alternatives that can be equilibrium outcomes of sophisticated voting over an amendment agenda when an unlimited number of amendments are allowed are exactly those alternatives that are the maximal elements of maximal chains. This set of alternatives is known as the *Banks set* and denoted by $B(T)$. Formally, this refinement of $UC(T)$ is defined as follows:

$$B(T) = \{x \in X : MH_T(x) \neq \emptyset\}.$$

3.4 Open Rules and Agendas

An *agenda* is an ordered subset of X. For any status quo $q \in X$, a *special rule* consists of an *initial agenda,* α_0, beginning with q, and a nonnegative number, $n \in \mathbf{Z}_+ \cup \infty$ denoting the number of amendments allowed to be "added" to α_0 on the floor. The set of all special rules, given the status quo q, is denoted $R(q)$. The notation $n = \infty$ denotes a truly open rule in the sense that the rule allows for an unlimited number of floor amendments, whereas $n = 0$ denotes a closed rule in the sense that no floor amendments are allowed. A special rule $r \in R(q)$, $r = (\alpha_0, n)$, is *unordered* if $\alpha_0 = q$ and *ordered* otherwise.[20] We denote the set of rules with $n = t$ by R^t, the set of unordered

[20]This representation of special rules is stylized in many ways of course. For example, we do not allow for the rule to specify an initial agenda "by section." Relaxing this restriction would allow for rules under which floor amendments may be considered "between" two or more of the items on the initial agenda, α_0. Additionally, the set of rules is restricted in the sense that they must use an amendment agenda (as opposed to king- or queen-of-the-hill procedures).

rules by R_U, and the set of ordered rules by R_S.[21]

For any agenda α and majority preference relation T, $\sigma(\alpha, T) \in \alpha$ denotes the sophisticated voting outcome of α given T (Shepsle and Weingast [1984a]). The ordered set of noninnocuous elements of α under T is denoted by $\eta(\alpha, T)$.[22] For any special rule r and majority preference relation T, the set of *potential outcomes achievable under* r is denoted by $\pi(r, T) \subset X$. This set is defined formally in the appendix. In words, it consists of all alternatives in x that are the sophisticated voting outcome for some agenda allowed under r. The next result characterizes this set under different special rules.

Theorem 1 *For any T, q, and special rule $r = (\alpha_0, n) \in R(q)$,*

1. *if $n = 0$, the set of potential outcomes is a singleton: $\pi(r, T) = \sigma(\alpha_0, T)$.*
2. *if $0 < n < \infty$, the set of potential outcomes is $\pi(r, T) = \left(\cap_{x \in \eta(\alpha_0, T)} T(x)\right) \cup \sigma(\alpha_0, T)$.*
3. *if $n = \infty$, the set of sophisticated voting outcomes is*

$$\pi(r, T) = B\left(\left(\cap_{x \in \eta(\alpha_0, T)} T(x)\right) \cup \sigma(\alpha_0, T)\right).$$

Proof: Proofs of all theorems and propositions are presented in the appendix. ∎

The theorem has two important implications. First, there are exactly three types of special rules with respect to the number of amendments allowed: closed ($n = 0$), open ($n = \infty$), and hybrid (n positive and finite). Second, the power of a rule lies in the specification of the initial agenda, α_0. Thus, from the Rules Committee's standpoint, the number of amendments allowed on the floor is meaningless unless it is zero. In

[21] Note that $R = R_U \cup R_S = \cup_{t=1}^{\infty} R^t$.

[22] The ordering of $\eta(\alpha, T)$ is induced by the ordering of α. Note as well that $q \in \eta(\alpha, T)$ for any α, given the assumption that an amendment agenda is used.

other words, there is no reason to restrict the number of amendments in any way other than reporting a closed rule.[23] More important to a policy-interested Rules Committee is the structuring of the agenda specified in the rule itself.

3.5 The Needing Relation

In this section, we define the *needing relation*, which characterizes the relationship between any point x and any other point y in terms of y's role in determining the eccentricity of x, $e_T(x)$. In words, if y is pivotal in the determination of $e_T(x)$, then x "needs" y. If x needs y, then this implies that there is some point z for which y is on the *unique* shortest chain from z to x. In a general and substantive sense, the needing relation determines whether one alternative x "needs" another, y, to be on the amendment agenda in order for x to be the sophisticated voting outcome under an open rule. Thus, while the notion of needing is defined in terms of an alternative's eccentricity, it has significant political implications for political agenda control.

Formally, for any tournament T, define the needing relation, N_T, as follows:

$$xN_Ty \Leftrightarrow \{z \in X_{-\{x,y\}} : \delta_{T_{-\{y\}}}(x,z) > \delta_T(x,z)\} \neq \emptyset.$$

Notice that xN_Ty holds only if $T_{-\{y\}}$ results in a longer shortest path from x to another alternative $z \neq y$. From this, it follows immediately that xN_Ty only if xTy (*i.e.*, x needs y only if x defeats y).

For any $Y \subseteq X$ and any $x \in Y$, we will write $N_{T_Y}(x)$ for the alternatives that need x in the subtournament induced on Y by T and $N^{-1}_{T_Y}(x)$ for the alternatives that x needs

[23] A note is in order at this point – from a formal standpoint, the conclusion that all positive but finite limitations on the number of floor amendments are equivalent relies on the presumption that there are enough distinct innocuous alternatives to "fill up" the agenda after floor consideration begins. Pun intended, this assumption seems innocuous: in reality, there are plenty of germane "loser" amendments to propose to any bill.

in T_Y. For any subset $Z \subseteq Y \subseteq X$, we will write $N_{T_Y}(Z) \equiv \{x \in Y : N_{T_Y}(x) \cap Z \neq \emptyset\}$ as the set of points in Y that need some alternative in Z and $N_{T_Y}^{-1}(Z) \equiv \cup_{x \in Z} N_{T_Y}^{-1}(x)$ as the set of points in Y that are needed by some alternative in Z.

3.5.1 Needing, Covering, and Agenda Manipulation

We now provide several results that illustrate the properties of the needing relation, N_T. The first states that no alternative x needs an alternative that is covered by x.

Proposition 1 *For any tournament T,*

1. xC_Ty *implies* $x \notin N_T(y)$ *and*

2. xN_Ty *implies there exists* $z \in X$ *such that* $xTyTzTx$.

The first conclusion of Proposition 1 states that no alternative needs an alternative that it covers. Substantively, this implies that agenda manipulation to achieve x as a sophisticated outcome need not involve the use of any alternative covered by x. This conclusion is indicative of the intuition behind the notion of "innocuousness," as utilized by Shepsle and Weingast [1984a]. If the sophisticated voting outcome of an agenda α is x, and the agenda contains some alternative y that is covered by x, the sophisticated voting outcome of the same agenda without y (*i.e.*, $\alpha' \equiv \alpha \setminus \{y\}$) is still x. In terms of agenda manipulation through the use of an ANS, Proposition 1 implies that an ANS can not be covered by the outcome that the agenda is "being manipulated" to achieve. Note that the proposition does *not* imply that the ANS must be an uncovered point.

The second conclusion of Proposition 1 (which is formally equivalent to the first conclusion) states that any alternative that needs another alternative is part of a 3-cycle in T. This conclusion makes clear the role of majority rule cycles in the power of agenda manipulation in this environment. If the majority preference relation is transitive, then manipulation of the order of voting (*e.g.*, through the pre-specification of an ANS) has no effect on which alternative will be chosen.[24] The possibility of majority rule cycles lies at the heart of the seminal objections of Riker [1980] to the structure induced equilibrium approach developed by Shepsle [1979]. Proposition 1 links the debate about structure induced equilibrium with the works of Miller [1980] and Shepsle and Weingast [1984b] (and others) by establishing the necessity of majority rule cycles for the effectiveness of procedural manipulations in general preference settings.

The next proposition states that any uncovered alternative x that needs another alternative y is no longer in the uncovered set if y is removed from the set of alternatives.

Proposition 2 *For any tournament T, xN_Ty implies that $x \notin UC(T_{-\{y\}})$.*

Proposition 2 is central to the understanding of ordered open rules in the following way. If, under a rule $= (\alpha_0, n)$, an alternative y is not in the uncovered set of the sophisticatedly relevant alternatives following the initial agenda α_0 (*i.e.*, $\sigma \cap_{x \in \eta(\alpha_0,T)} T(x)$, it necessarily can not be the sophisticated voting outcome under an open rule). The following corollary makes this point in a more direct fashion.

Corollary 1 *For any tournament T, status quo q, and open rule $r = (\alpha_0, \infty)$, $xN_{T(q)}y$ implies that $x \in \pi(r, T)$ only if either*

1. $y \in \eta(\alpha_0, T)$ *or*

2. $y \in \left(\cap_{x \in \eta(\alpha_0,T)} T(x)\right)$.

[24] Clearly, other forms of agenda manipulation (*e.g.*, rules and procedures that limit the set of alternatives that may be considered) will in general still play a role in such settings. This form of manipulation has been extensively studied elsewhere (Romer and Rosenthal [1978], Denzau and Mackay [1983], *etc.*) and is beyond the scope of this paper.

The corollary states that, if an alternative x needs another alternative y (given the status quo q), then x can be chosen under an open rule only if y has not been rendered innocuous by the initial agenda specified in the rule, α_0. As the corollary's statement makes clear, the non-innocuousness of y can be preserved by a rule in which no element $z \in \alpha_0$ defeats y unless y occupies a position on α_0 that precedes z. Assuring that an element y that is needed by x is not made innocuous by the rule can be guaranteed by placing y as the first element of α_0: in other words, designing the rule r so that y is considered as the bill itself. Thus, under an open rule, the bill itself will not be the ultimately chosen alternative (unless the bill is a Condorcet winner). Nevertheless, including it on the voting agenda may be central to the procedural manipulation of collective choice. Specifically, the presence of the bill on the amendment agenda will itself render the subsequent proposal of some other alternatives innocuous.

According to Theorem 1, the predicted outcomes under an open rule are determined by the Banks set. Accordingly, the next proposition of the section applies the needing relation to the Banks set. In words, the proposition states that any alternative, y, that is needed by another element, x, is a member of every maximal chain in which x is top-ranked. In other words, if x is in the Banks set and x needs y, then every agenda that justifies the membership of x in the Banks set contains y. Substantively, this proposition states that the needing relation describes the procedural maneuvers required to achieve outcomes through sophisticated voting under an open rule.[25]

Proposition 3 *For any tournament T and $x, y \in X$, if $x N_T y$, then $y \in Y$ for all $Y \in H_T(x)$.*

[25] Proposition 3 implies that any alternative x that is needed by an alternative y in the Banks set (*i.e.*, $y N_T x$ & $y \in B(T)$) is "Banks-necessary" for y (Moser, Patty, and Penn [2006]).

3.6 Open Rules, Procedures, and Majoritarianism

The results above illustrate the importance of needed alternatives in the manipulation of collective choice under an open rule. The next example illustrates that alternatives which can not be selected under an open rule may nevertheless be essential to implement some alternatives under such a rule. In other words, majority rule under an open rule may be manipulated through the use of alternatives that are not in the Banks set. This fact suggests a "counter-majoritarian" aspect of ordered open rules. In addition to raising philosophical questions about the proper design of collective choice procedures (Dummett [1984], Schwartz [1990]) this fact raises an important question about the way in which we examine and evaluate the determinants of agenda setting (and accordingly policy-making) power in legislative bodies such as the U.S. House of Representatives.[26]

Our results show that, while sophisticated voting over an amendment agenda is always majoritarian in the sense that the final chosen outcome is always weakly preferred to the initial (*i.e.*, status quo) policy, this is a very weak notion of "majoritarian." In particular, this notion allows the selection of alternatives that are not in the uncovered set. A strengthened notion of majoritarianism would require that an alternative be selected only if it can be chosen through an open rule in which amendments are permitted only if those amendments can themselves be selected under an open rule. This requirement can be thought of as ruling "out of order" any amendment that can not itself be the final selected outcome under an open rule. This notion of majoritarianism may be defined formally for any tournament T and status quo q as requiring

[26] A list of relevant recent work, in addition to that cited earlier in the paper, would include Bawn [1998], Krehbiel [1997, 1999], Krehbiel and Meirowitz [2002], Theriault [2005], Lebo, McGlynn, and Koger [2007], Patty [2007], and Wiseman and Wright [2007]).

that an element of the set $\mu(T,q)$ be selected, where this set is defined as:,

$$x \in \mu(T,q) \Leftrightarrow MH_{T(q)}(x) \cap 2^{(B(T(q)) \cup \{q\})} \neq \emptyset.$$

For any tournament T and status quo q, we will refer to alternatives in $\mu(T,q)$ as satisfying *μ-majoritarianism* or, more simply, as μ-majoritarian. In words, an alternative is μ-majoritarian is an alternative that can be implemented through the use of an amendment agenda containing only the status quo and elements of the Banks set of $T(q)$.[27] Viewed slightly differently, the alternatives in $B(T(q)) \setminus \mu(T,q)$ – *i.e.*, those alternatives that can be selected under an open rule but are not μ-majoritarian – are achievable under an open rule only through the proposal of one or more amendments that are themselves not in the Banks set of $T(q)$. Such amendments represent, in a very specific and unambiguous sense, manipulation of the collective choice process.

The next corollary, which follows from Proposition 3, formally states that a necessary condition for an alternative to satisfy μ-majoritarianism is that it not need any alternative not in the Banks set of $T(q)$.

Corollary 2 *For any tournament T, status quo q, and alternative $y \notin B(T(q))$, $xN_{T(q)}y$ implies $x \notin \mu(T,q)$.*

The corollary has the following implication about the relationship between the needing relation and μ-majoritarianism: any alternative needed by a policy that satisfies μ-majoritarianism must itself be achievable under some open rule. This fact clarifies the tight link between the notion of μ-majoritarianism and agenda manipulation through

[27] Note that this is not the strongest version of this kind of requirement. One could extend the definition in an iterative fashion by, for example, defining $\mu^2(T,q)$ as the set of alternatives that are top-ranked in maximal chains containing only the status quo q and elements of $\mu(T,q)$. This could then be extended to create analogous definitions of $\mu^3(T,q)$, $\mu^4(T,q)$, and so forth. We do not examine these extensions of $\mu(T,q)$ in this paper because – while interesting from mathematical and philosophical standpoints, the extensions of μ lose any strong link with majoritarianism.

the structuring of open rules. The next result demonstrates that some alternatives will require agenda manipulation in order to be selected under an open rule.

Proposition 4 *For any set X containing at least six elements, there exist tournaments with $\mu(T, q) \neq B(T(q))$.*

Proposition 4 is interesting because it illustrates that not all outcomes achievable through an open rule are "created equal." In particular, some alternatives require procedural finagling in the sense that effectively unselectable outcomes must be included on the agenda if the alternative is to be selected. The next result states that ruling out such alternatives (for either positive or normative reasons) is acceptable in the sense that there is always at least one alternative that does not require such manipulation to be chosen under sophisticated voting. Furthermore, the theorem precisely characterizes the set of outcomes that do not require procedural manipulations to be selected under an open rule.

Theorem 2 *For any nonempty set X, status quo $q \in X$, and any $T \in \mathcal{T}_{|X|}$, $\mu(T, q) \neq \emptyset$. Specifically, $\mu(T, q) = B(B(T(q)))$.*

Theorem 2 is particularly interesting in light of the characterization of μ-majoritarian outcomes. In words, the outcomes that do not require procedural manipulation to be selected under an open rule are precisely those that might be selected through open rule if the legislature initially threw out all alternatives that can not be selected under an open rule. Formally, the theorem demonstrates that this solution concept, the "Banks set of the Banks set" (or the Banks^2 set), has an interesting (and to our knowledge, previously unnoticed) procedural justification. In addition to being substantively interesting, the result highlights a technical feature of the Banks^2 set: each element in this solution concept is supported by an agenda that is maximally stable within both the Banks set *and* the original set of alternatives.

It should be noted that selection of an alternative not in the Banks^2 set under an open rule requires the placement of at least element outside of the Banks set on the voting agenda. However, it is not the case that an element outside of Banks^2 set needs – in the formal sense – an alternative outside of the Banks set. Needing an alternative outside of the Banks set is a sufficient condition to not be in the Banks^2 set and, accordingly by Corollary 2, a sufficient condition for the alternative in question to not satisfy μ-majoritarianism.

3.7 Needing, Agenda Control, and Procedures

The results in this section jointly raise the question of what solution concept, if any, selects the set of outcomes that do not require agenda manipulation (as conceived here in terms of requiring a precise specification of the final voting order) to be achieved. Such a solution concept would have desirable positive and normative properties. On the positive side, such a solution concept would identify outcomes that are in a strong sense achievable regardless of the preferences of a "procedure-setter" as exemplified by the Rules Committee in the U.S. House of Representatives. Clearly, if a rule r can be specified by a subset of the legislature, the prediction of what outcome or outcomes *will* occur will depend upon the actions (and accordingly preferences) of the individual or individuals with the power to propose rules. Nonetheless, elements of a solution concept that do not require agenda manipulation would remain feasible (again, in a strong sense) even if the identity or identities of those with the power to propose rules were changed.

In a normative sense, the discussion of μ-majoritarianism highlights the fact alternatives that do not require agenda manipulation are consistent with a strong and internally consistent notion of "majority will." Of course, it is well known[28] that the notion of majority will is complex and fraught with ambiguities. However, even in the absence of a Condorcet winner, defining "inconsistency with any sensible notion of majority will" can often be less difficult to work with.[29] The notion of μ-majoritarianism – applied as a notion of "unmanipulated majority will" – does not necessarily select a unique alternative. However, as illustrated above, it does eliminate some alternatives that are not eliminated by other solution concepts.

It is interesting to note that the equivalence established in Theorem 2 between μ-majoritarianism and the Banks^2 set is also consistent with the institutional reality in the U.S. House of Representatives. In particular, if one presumes that the Rules Committee commits itself to report an open rule, the effective set of alternatives over which the members of the committee are bargaining are the elements of the Banks set (given the status quo q and the majority preference relation for the entire chamber, T). The standing rules of the House dictate that committees use the same procedures as are used in the House.[30] Accordingly, the decision about which open rule to report reduces to a decision made through the use of an amendment agenda (under an open rule) about which alternative in the Banks set to implement. Thus, presuming for parsimony that the Rules Committee has the same majority preference relation as the House as a whole, the set of outcomes that might be chosen are exactly equal to the Banks^2 set. Hence, without directly including the Rules Committee in the model, *per se*, we nonetheless reach a conclusion consistent with its inclusion.

In pursuit of a solution concept that selects alternatives that do not require agenda manipulation involving alternatives that can not themselves be chosen, we define the

[28]See, for example, Condorcet [1785], Arrow [1951], Plott [1967], McKelvey [1976], Schofield [1978], and Rubinstein [1979], among others.

[29]This point is illustrated by the notion of Pareto optimality (Pareto [1971 / 1906], Wittman [1989]).

[30]*C.f.* House Rule XI, clause 1(a).

notion of *independence*. Motivated by Proposition 2, Proposition 3, and Corollary 2, this notion applies the needing relation to define a necessary condition for a concept of majority choice that does not require agenda manipulation to be implemented through sophisticated voting over amendment agendas. This is a desirable property insofar as the use of amendment agendas is frequently at least a "fall-back" or default option in many collective choice institutions (Rasch [2000] and Tsebelis [2002]).

Informally speaking, a solution concept is a function, S, that selects (sets of) alternatives based on the majority preference tournament T.[31] Accordingly, a solution concept is independent if it does not select any alternative that needs (in the sense of the binary relation $N_{S(T)}$) some alternative not selected by the solution concept. The following states this definition formally.

Definition 2 (Independence) *A solution concept S satisfies* independence *(or,* is independent*) if, for any tournament T,*

$$N_T^{-1}(S(T)) \subseteq S(T).$$

In pursuit of a solution concept that refines itself as much as possible, Dutta [1988] defines the *minimal covering set*, which is a refinement of the uncovered set. The minimal covering set has attracted less attention than it deserves: it is not only a refinement of the uncovered set, but it also turns out to have a very appealing stability property related to the needing relation and, accordingly, to the notion of independence defined above. The minimal covering set's definition is based upon the notion of a *covering set*. A covering set for a finite tournament is a subset of the alternatives such that any alter-

[31] Formally, a function $S : \mathcal{T} \to 2^{X(T)}$ is a *tournament solution concept* if (1) $S(T) \neq \emptyset$ for all $T \in \mathcal{T}$, (2) selects a Condorcet winner (and only the Condorcet winner) of T if one exists and (3) is invariant to tournament isomorphisms (*i.e.*, relabelings of the alternatives). See Laslier [1997] for a definition of tournament isomorphisms and a detailed discussion of tournament solution concepts.

native outside of the subset is covered by an alternative in the subset if that alternative is individually added to the subset.

Definition 3 *For any tournament T, a set $Y \subseteq X$ is a* covering set *of T if $x \in C(T_{Y\cup\{x\}})$ for all $x \in X \setminus Y$.*

For any tournament T, let $\mathcal{C}(T)$ denote the set of covering sets of T. The next result states that the needing relation is closed under the intersection of covering sets.

Proposition 5 *For any tournament T, any pair of nested covering sets $Y, Z \in \mathcal{C}(T)$ with $Y \subseteq Z$, any uncovered alternative $x \in UC(T_Y)$, and any alternative $y \in Y$,*

$$xN_{T_Z}y \Rightarrow xN_{T_Y}y.$$

Formally, Proposition 5 ensures that the need for agenda manipulation to achieve some outcome (in the statement of the proposition, this alternative is represented by x) is preserved in the face of the elimination of alternatives from the space by the covering relation. This fact is intimately tied to the relationship between needing, as characterized by N_T, and sophisticated voting. To see this, note that an alternative x not being in a covering set (*i.e.*, $X \setminus \{x\}$ is a covering set), then it follows that there exists some other alternative that x cannot defeat through sophisticated voting over *any* amendment agenda. This fact motivates the use of covering sets to winnow the set of alternatives in order to refine the set of predicted outcomes. Along these lines, Dutta [1988] demonstrates that $\mathcal{C}(T)$ possesses a unique element that is minimal with respect to set inclusion or, equivalently, that $\mathcal{C}(T)$ is closed with respect to intersection.[32] It follows that one may unambiguously define the minimal covering set, $MC(T)$, as

[32] See also the presentation and development of Laslier [1997].

follows:

$$MC(T) = \{x \in X : Y \in \mathcal{C}(T) \Rightarrow x \in Y\}.$$

The combination of Dutta [1988] and Proposition 5 implies the following result, which states that the minimal covering set is independent.

Proposition 6 *For any tournament T, $x \in MC(T)$ and $y \in X$, $xN_Ty \Rightarrow y \in MC(T)$.*

Finally, Theorem 3 summarizes the independence of the four solution concepts discussed in this paper. The importance of Theorem 3 lies in the relationship between each of the solution concepts and the role of agenda control in guaranteeing that outcomes in that solution concept can be achieved under an open rule. Outcomes in independent solution concepts can be achieved through consideration of agendas containing *only* outcomes consistent with the solution concept itself. Accordingly, the elements used in the agenda manipulations required to implement an independent solution concept are themselves consistent with the solution concept itself.

Theorem 3

1. *TC is independent.*

2. *UC is not independent.*

3. *B is not independent.*

4. *MC is independent.*

4 Conclusions

The theory presented here enriches our understanding of the impact of legislative procedure on policy outcomes by extending the graph-theoretic approach to collective choice. The results extend the insights of Ordeshook and Schwartz [1987] and further establish the theoretical relevance of the notion of eccentricity of an alternative within the graph of the tournament in which it is embedded. Propositions 1-2 help describe the needing relation, clarify its relationship with the notion of covering, and accordingly elucidate the intimate relationship between needing and the possibility and necessity of agenda control even when consideration is governed by an open rule.

Proposition 3 establishes the role of the needing relation in the stability of the Banks set. In particular, a key consideration in attempting to eliminate items from the Banks set is to identify which alternatives in the Banks set "need" any other alternative. Since neither the Banks set nor the uncovered set are independent (Theorem 3), there exist tournaments in which one can eliminate an alternative from the Banks set through elimination of a point outside the uncovered set.[33] Substantively, the results presented above illustrate the importance and operation of agenda control even when floor consideration is governed by an apparently open rule, since sophisticated voting is especially sensitive to the first things proposed (*i.e.*, the last things voted upon) when an amendment agenda is used. In other words, the "elimination" of (covered or uncovered) alternatives can be accomplished through the specification of an ordered open rule in which one or more amendments are given priority in recognition on the floor.

The characterization of the minimal covering set, $MC(T)$, in Proposition 6 serves as the principal motivation behind the definition of independence and Theorem 3. Since the notion of independence represents a necessary condition for a solution concept to be invariant to the manipulation of amendment agendas, these results suggest that the minimal covering set represents an attractive alternative for "institution-free" consid-

[33]This is shown in Moser et al. [2006].

erations of agenda manipulation.[34] Furthermore, the results presented here – taken as a whole – describe an intimate relationship between the majoritarian nature of legislative procedures and the independence of the set of outcomes that the procedures may yield. While there is much work to be done about the details of procedures and and collective decisions, these results draw a connection between the "nitty-gritty" aspects of legislative decision making and bigger formal and philosophical questions about the importance of institutions in collective choice.[35]

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[34] This conclusion is similar in spirit to the analysis in McKelvey [1986].

[35] With respect to the bigger questions, see, among others, Riker [1980, 1986], Tullock [1981], Ordeshook and Schwartz [1987], Krehbiel [1999], and Schickler [2002].

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A Proofs of Numbered Propositions and Theorems

A.1 Theorem 1

The set of all finite agendas beginning with an initial agenda α_0 is denoted by $A(\alpha_0)$. For any integer $n \geq 0$, the set of all agendas beginning with an initial agenda α_0 and containing exactly n alternatives following α_0 is denoted by $A^n(\alpha_0)$.[36] The set $A^\infty(\alpha_0) \equiv MH(T) \cap A(\alpha_0)$ is defined to be the set of all externally stable chains under T beginning with α_0. The set of all finite amendment agendas, given the status quo $q \in X$, is $A(\{q\})$. Given the majority preference T, the status quo $q \in X$, and an alternative $x \in X$, the set of finite amendment agendas yielding x as the sophisticated outcome is denoted by

$$V(x, T, q) \equiv \{\alpha \in A(\{q\}) : \sigma(\alpha, T) = x\}.$$

The set of potential outcomes following from a rule $r = (\alpha_0, n) \in R(q)$, given T, is defined as follows:

$$\pi(r, T) = \{x \in X : A^n(\alpha_0) \cap V(x, T, q) \neq \emptyset\}.$$

(Note that the first element of r must be q, implying that we do not need to include q as an argument for π.)

Assumption 1 *There exists a subset of X, ι, with $|\iota| = \infty$, such that qTy for all $y \in |\iota|$.*

Assumption 1 is a technical assumption that guarantees that any length of agenda can be "filled out" with proposals that are defeated by the status quo, q. Descriptively

[36] Thus, $A^0(\alpha_0) = \{\alpha_0\}$ and $A(\alpha_0) = \cup_{n=0}^{\infty} A^n(\alpha_0)$.

speaking, this assumption is plausible. It is unnecessary if one allows for repetitious consideration of an amendment.

Theorem 1. For any T, q, and special rule $r = (\alpha_0, n) \in R(q)$,

1. if $n = 0$, the set of potential outcomes is a singleton: $\pi(r, T) = \sigma(\alpha_0, T)$.
2. if $0 < n < \infty$, the set of potential outcomes is $\pi(r, T) = \left(\cap_{x \in \eta(\alpha_0, T)} T(x)\right) \cup \sigma(\alpha_0, T)$.
3. if $n = \infty$, the set of sophisticated voting outcomes is

$$\pi(r, T) = B\left(\left(\cap_{x \in \eta(\alpha_0, T)} T(x)\right) \cup \sigma(\alpha_0, T)\right).$$

Proof: 1. Follows directly from Shepsle and Weingast [1984a] (see also McKelvey and Niemi [1978]).

2. By construction. Fix $0 < n < \infty$ and $r \in R(q)$. To see that $\sigma(\alpha_0, T) \in \pi(r, T)$, consider any agenda α in which every floor amendment is an element of ι (Assumption 1): $\alpha \setminus \alpha_0 \subset \iota$. The definition of ι implies that $\sigma(\alpha, T) = \sigma(\alpha_0, T)$. For any element $y \in \cap_{x \in \eta(\alpha_0, T)} T(x)$, construct an agenda α with $\alpha \setminus \alpha_0 \setminus \iota = \{y\}$: it follows that $\sigma(\alpha, T) = \sigma(\alpha_0 \cup \{y\}, T) = y$. Finally, for any element $y \notin \cap_{x \in \eta(\alpha_0, T)} T(x)$, it follows from Shepsle and Weingast [1984a] that for any $\alpha \in A^n(\alpha_0)$, $v(\alpha, T) = v(\alpha \setminus \{y\}, T)$, implying that $y \notin \pi(r, T)$.

3. Follows immediately from Shepsle and Weingast [1984a] and Banks [1985]. ∎

A.2 Propositions 1-6

Proposition 1. For any tournament T,

1. $xC_T y$ implies $x \notin N_T(y)$ and

2. xN_Ty implies there exists $z \in X$ such that $xTyTzTx$.

Proof: 1. $xN_Ty \Rightarrow \exists z \in X$ s.t. $yTzTx$, which implies that y can beat x in two-steps. Thus, x does not cover y.

2. Since xN_Ty implies that xTy and x does not cover y (by point 1, above), it follows that $\delta_T(x, y) = 2$. ∎

Proposition 2. For any tournament T, xN_Ty implies that $x \notin UC(T_{-\{y\}})$.

Proof: Fix T and consider any x, y such that xN_Ty. There are two cases to consider (1) $x \notin UC(T)$ and (2) $x \in UC(T)$. Considering the case of $x \notin UC(T)$, first note the following facts:

i. if $x \notin UC(T)$, then $e_T > 2$ and

ii. xN_Ty implies that $e_{T_{-\{y\}}}(x) \geq e_T(x)$.

Combining these facts, it follows that $x \notin UC(T)$ implies that $x \notin UC(T_{-\{y\}})$.

Accordingly, suppose that $x \in UC(T)$. Then by Fact 1, $\delta_T(x, z) = 2$ for all $z \in X_{-x}$ and $\delta_{T_{-\{y\}}}(x, z')$ for some $z' \in X_{-\{x,y\}}$, implying that $e_{T_{-\{y\}}}(x) > 2$. Therefore, $x \notin UC(T_{-\{y\}})$, as was to be shown. ∎

Proposition 3. For any tournament T and $x, y \in X$, if xN_Ty, then $y \in Y$ for all $Y \in H_T(x)$.

Proof: Fix T and consider first any $x \in B(T)$ such that $N^{-1}(x) \neq \emptyset$.[37] (We deal with the case of $x \notin B(T)$ below.) Take any chain $Y \in H_T(x)$. By the definition of a maximal chain,

1. xTz for all $z \in Y \setminus \{x\}$ and

[37] If no such alternative exists, the result holds trivially.

2. either xTw or $xTzTw$ for all $w \notin Y$ and some $z \in Y$.

The presumption that $y \in N_T^{-1}$ implies that there exists some $w \notin Y$ (because wTx) such that $xTzTw$ implies that $z = y$. Since Y is arbitrary, this implies that $y \in Y$ for all $Y \in H_T(x)$. Finally, recall that $H_T(x) \neq \emptyset$ if and only if $x \in B(T)$, so that the result trivially holds for any $x \notin B(T)$, concluding the proof. ∎

Proposition 4. For any set X containing at least six elements, there exists a tournament T and status quo policy $q \in X$ with $\mu(T, q) \neq B(T(q))$.

Proof: Consider the tournament T_2 in Section C with status quo q. This example contains exactly six alternatives, which establishes the result for all tournaments of greater order because this example may be extended to $k > 6$ elements by considering tournaments in which a subset X' containing exactly six elements exists (with the subtournament $T_{X'}$ being given by T_1) and every element in X' defeating every element outside of X'. ∎

Proposition 5. For any tournament T, any pair of nested covering sets $Y, Z \in \mathcal{C}(T)$ with $Y \subseteq Z$, any uncovered alternative $x \in UC(T_Y)$, and any alternative $y \in Y$,

$$xN_{T_Z}y \Rightarrow xN_{T_Y}y.$$

Proof: The supposition that Y and Z are both covering sets with $Y \subset Z$ implies that, for all $z \in X \setminus Y$, $z \notin UC(Y \cup \{z\}) \cap UC(Z \cup \{z\})$.

If $z \in Y$, then the result clearly holds by the supposition that $x \in UC(Y, T_Y)$ and the definition of the uncovered set.

If $z \notin Y$, then the construction of z implies that $T(z) \cap T^{-1}(x) \cap Y = \{y\}$, and the fact that Y is a covering set implies that there exists $w \in Y$ such that $wC_{T_{Y \cup \{z\}}}z$, implying that $T(w) \cap Y \subseteq T(z) \cap Y$, so that $T(w) \cap T^{-1}(x) \cap Y \neq \emptyset$ implies that

$T(w) \cap T^{-1}(x) \cap Y = \{y\}$. If $T(w) \cap T^{-1}(x) \cap Y = \emptyset$, then $x \notin UC(T_Y)$. Accordingly, $T(w) \cap T^{-1}(x) \cap Y = \{y\}$, which implies that x needs y to defeat w in Y: $xN_{T_Y}y$, as was to be shown. ∎

Proposition 6. For any tournament T, $x \in MC(T)$ and $y \in X$, $xN_Ty \Rightarrow y \in MC(T)$.

Proof: Fix a tournament T and consider any $x \in MC(T)$ and $y \in X$ such that xN_Ty.

xN_Ty implies that there exists $z \in X$ such that $T^{-1}(x) \cap T(z) = \{y\}$. Accordingly, $zC_{T_{X'}}x$ for any $X' \subseteq X_{-y}$ with $z \in X'$. Consider two cases in order: (i) $z \in MC(T)$ and (ii) $z \notin MC(T)$.

Case (i): $z \in MC(T)$. Clearly, $UC(MC(T)) = MC(T)$. Thus, if $z \in MC(T)$, then it follows that $y \notin MC(T) \Rightarrow x \notin MC(T)$. Thus, $y \in MC(T)$, as was to be shown.

Case (ii): $z \notin MC(T)$. The definition of $MC(T)$ and the supposition $z \notin MC(T)$ jointly imply that there exists $w \neq x$ such that $wC_{T_{MC(T)\cup\{z\}}}z$. The covering relation is transitive (Fishburn [1977], Miller [1980]). Thus, since $zC_{T_{X'}}x$ for any $X' \subseteq X_{-y}$ with $z \in X'$, it follows immediately that $wC_{T_{X''}}x$ for any $X'' \subseteq X_{-y}$ with $w \in X''$. As with case (i), above, it now follows immediately from $UC(MC(T)) = MC(T)$ that $y \in MC(T)$. ∎

A.3 Theorem 2

Theorem 2. For any nonempty set X, status quo $q \in X$, and any $T \in \mathcal{T}_{|X|}$, $\mu(T, q) \neq \emptyset$. Specifically, $\mu(T, q) = B(B(T(q)))$.

Proof: Fix $X \neq \emptyset$, a status quo $q \in X$, and a majority preference relation T on X. Nonemptiness of $B(B(T(q)))$ follows from the nonemptiness of $B(T)$ (Banks [1985]). To first show that $B(B(T(q))) \subseteq \mu(T, q)$, consider any element $x \in B(B(T(q)))$ and let $\alpha_x \subset B(T(q))$ be a chain that is externally stable in $B(T(q))$ (and therefore, $\alpha_x \in$

$2^{(B(T(q))\cup\{q\})}$). If α_x is externally stable in $T(q)$, then $x \in \mu(T,q)$. If, on the other hand, α_x is not externally stable in $T(q)$, then there exists some extension of α_x, denoted by α'_x, such that the first $|\alpha_x|$ elements of α'_x are identical to α_x and α'_x is externally stable with respect to $T(q)$. By the definition of the Banks set, the top-ranked element of α'_x, denoted by x', is an element of the Banks set of $T(q)$ (*i.e.*, $x' \in B(T(q))$), implying that $x'Ty$ for all $y \in \alpha_x$ and contradicting the supposition that α_x is externally stable in $B(T(q))$. Accordingly, $\alpha_x \in MH_{T(q)}(x)$ and $\alpha_x \in 2^{(B(T(q))\cup\{q\})}$, implying that $x \in \mu(T,q)$ or, more generally, $B(B(T(q))) \subseteq \mu(T,q)$. This also implies $\mu(T,q) \neq \emptyset$.

To finish the proof, we show that $\mu(T,q) \subseteq B(B(T(q)))$ as follows. Consider any $x \in \mu(T,q)$. By definition, we can choose an agenda $\beta_x \in MH_{T(q)}(x) \cap 2^{(B(T(q))\cup\{q\})}$. If the top-ranked element of this agenda, x, is not in $B(B(T(q)))$, then we immediately reach a contradiction, as $\beta_x \not\subseteq 2^{(B(T(q))\cup\{q\})}$. Therefore, $x \in B(B(T(q)))$, so that $\mu(T,q) \subseteq B(B(T(q)))$ and $B(B(T(q))) \subseteq \mu(T,q) \subseteq B(B(T(q)))$, implying that $\mu(T,q) = B(B(T(q)))$, as was to be shown. ∎

A.4 Theorem 3

Theorem 3.

1. TC is independent.

2. UC is not independent.

3. B is not independent.

4. MC is independent.

Proof:

1. TC *is independent.* Suppose otherwise: xN_Ty with $x \in TC(T)$ and $y \notin TC(T)$. Then there exists z such that $T(z) \cap T^{-1}(x) = \{y\}$. Since $TC(T)$ is T-retentive and zTx, then $z \in TC(T)$, which implies that $y \in TC(T)$.

2. UC *is not independent.* Consider the tournament T_1 in Section B.

3. B *is not independent.* Consider the tournament T_1 in Section B.

4. MC *is independent.* Proposition 6, above. ∎

B Tournament T_1

Consider the set $X = \{a, b, c, d, e\}$, with the following majority preference relation:

$$T_1 = \begin{array}{l} aTb,\ aTe \\ bTc,\ bTd,\ bTe \\ cTa,\ cTe \\ dTa,\ dTc \\ eTd \end{array}$$

1. $N_{T_1}^{-1}(c) = \{e\}$

2. $UC(T_1) = B(T_1) = \{a, b, c, d\}$

3. $UC^2(T_1) = UC^\infty(T_1) = \{a, b, d\}$

C Tournament T_2

Consider the set $X = \{a, b, c, d, e, q\}$, with the following majority preference relation:

$$T_2 = \begin{array}{l} aTb,\ aTe,\ aTq \\ bTc,\ bTd,\ bTe,\ bTq \\ cTa,\ cTe,\ cTq \\ dTa,\ dTc,\ dTq \\ eTd,\ eTq \end{array}$$

Note that q is a Condorcet loser and the top-cycle of T is $TC(T_2) = \{a, b, c, d, e\}$, with $T_{TC(T_2)} = T_{T_2(q)} = T_1$ (where T_1 is defined in Section B). Accordingly, let T_1 denote $T_{T_2(q)}$ and note that, if q is the status quo, then $N_{T_1}^{-1}(c) = N_{T_1}^{-1}(c) = \{e\}$. Furthermore, $UC(T_1) = B(T_1) = \{a, b, c, d\}$, so that

$$\mu(T_2, q) = \{a, b, d\}.$$

To see this, note that

$$\begin{aligned} MH_{T_1}(a) &= \{(a, b, q), (a, b, e, q)\} \\ MH_{T_1}(b) &= \{(b, c, q), (b, d, q), (b, d, c, q), (b, e, d, q), (b, c, e, q)\} \\ MH_{T_1}(c) &= \{(c, a, e, q)\} \\ MH_{T_1}(d) &= \{(d, a, q), (d, c, a, q)\} \\ MH_{T_1}(e) &= \emptyset. \end{aligned}$$

Accordingly,

$$
\begin{aligned}
MH_{T_2(q)}(a) \cap 2^{(B(T_2(q)) \cup \{q\})} &= \{(a, b, q)\} \\
MH_{T_2(q)}(b) \cap 2^{(B(T_2(q)) \cup \{q\})} &= \{(b, c, q), (b, d, q), (b, d, c, q)\} \\
MH_{T_2(q)}(c) \cap 2^{(B(T_2(q)) \cup \{q\})} &= \emptyset \\
MH_{T_2(q)}(d) \cap 2^{(B(T_2(q)) \cup \{q\})} &= \{(d, a, q), (d, c, a, q)\} \\
MH_{T_2(q)}(e) \cap 2^{(B(T_2(q)) \cup \{q\})} &= \emptyset,
\end{aligned}
$$

so that

$$\mu(T_2, q) = \{a, b, d\} \subsetneq B(T_2(q)).$$