Constituencies and the Allocation of Private Activity Municipal Bond Cap Authority

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Background

- Federal government limits the total volume of tax exempt borrowing by private entities in a state and calendar year
- Tax exemption allows lower borrowing costs, and creates potential profits for private entities
- Most borrowing falls into five categories

Industrial Development
Utilities (privately owned)
Mortgage Revenue
Multifamily housing
Student Loans

Objectives

- Estimate the relative influence of the competing constituencies on the level of allocation
- Estimate the relative transfer of rents to public officials from the competing constituencies
- Characterize the connection between the scale of the economy in relevant industries and private activity borrowing

Existing Literature

- Very little on private activity authority allocation
  - Zimmerman 1990
  - Kenyon 1991
  - Temple 1993
- Considerable literatures on political allocation of government spending, infrastructure investment, tax burdens, etc.
  - Cadot, Roller & Stephan (2006)
- Overviews
  - Grossman & Helpman (2001)
  - Persson & Tabellini (2000)

Simple Model of Allocation of Authority Across Groups

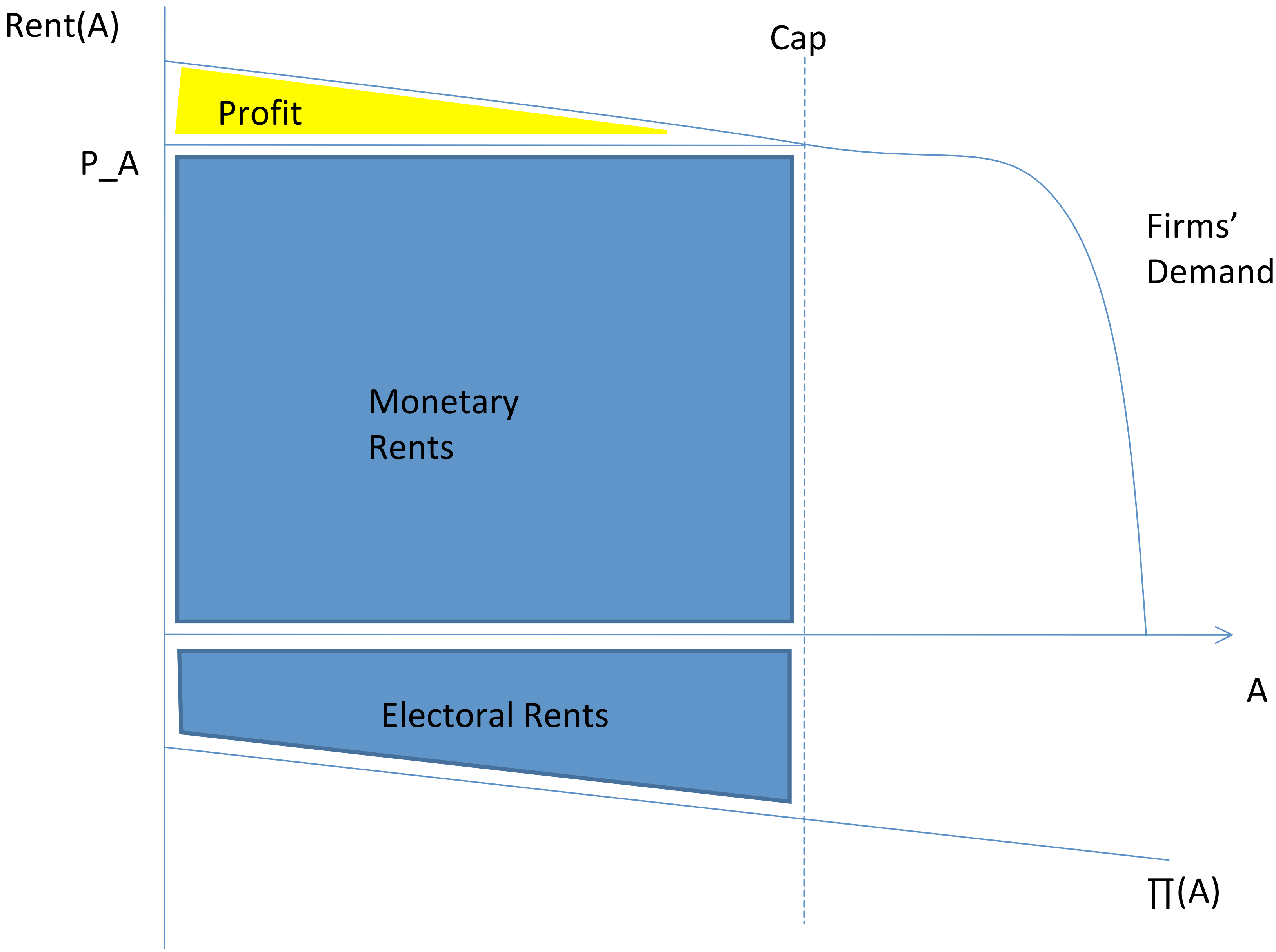
A – Total tax exempt private activity borrowing
∏ - political value of A
P_A – Per unit transfer (price) from firm to official for borrowing authority
Y – firm's output
P_y – market price of firm's output
L – firm's other inputs
P_L – price of firm's other input
r – tax exempt interest rate
a – an administrative cost
M – bond maturity
π – firm's profit
ρ – discount rate

The Public Official's Utility

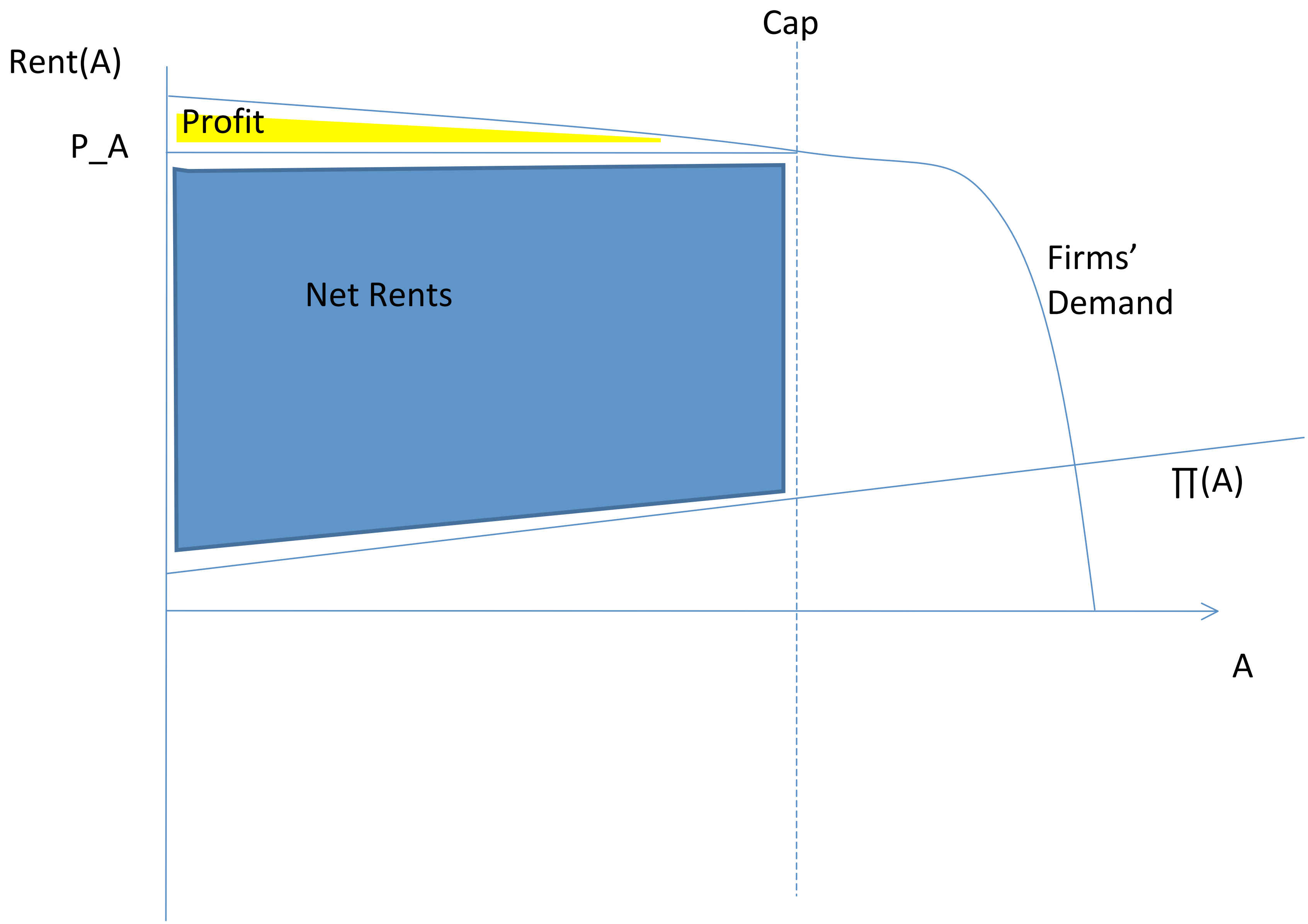
$$U(A) = \Pi(A) + P_A A$$
$$\frac{\partial U}{\partial A} = \frac{\partial \Pi}{\partial A} + P_A$$

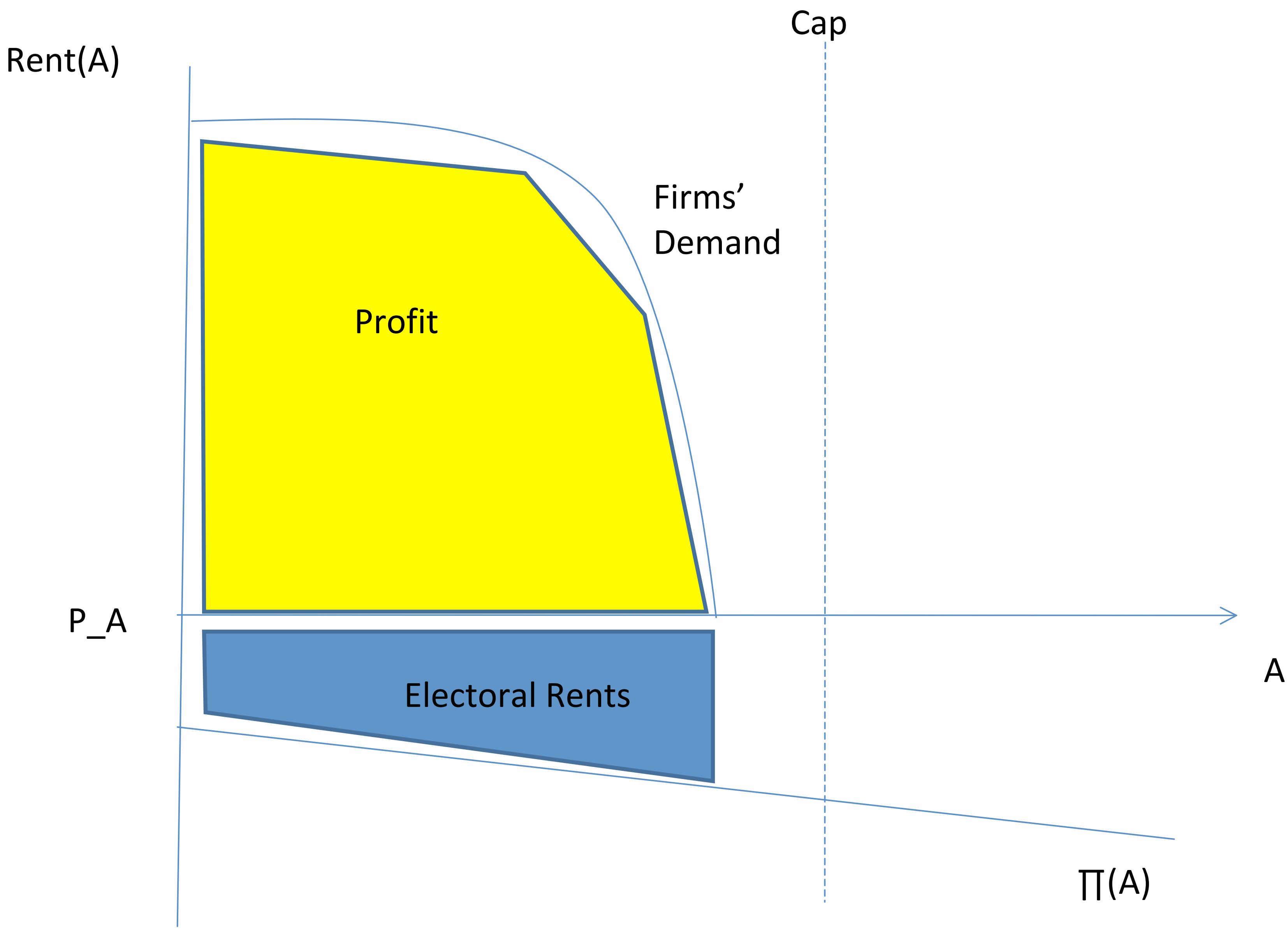
The Representative Firm's Profit in Present Value

$$PV(\pi) = \sum_x^M \frac{(P_y y - P_L L - rA - a)}{(1+\rho)^x} - \frac{A}{(1+\rho)^M} - P_A A$$
$$\frac{\partial PV(\pi)}{\partial A} = \sum_x^M \frac{P_y \frac{\partial y}{\partial A}}{(1+\rho)^x} - \sum_x^M \frac{r}{(1+\rho)^x} - \frac{1}{(1+\rho)^M} - P_A$$
$$\sum_x^M \frac{P_y \frac{\partial y}{\partial A}}{(1+\rho)^x} = \sum_x^M \frac{r}{(1+\rho)^x} + \frac{1}{(1+\rho)^M} + P_A$$

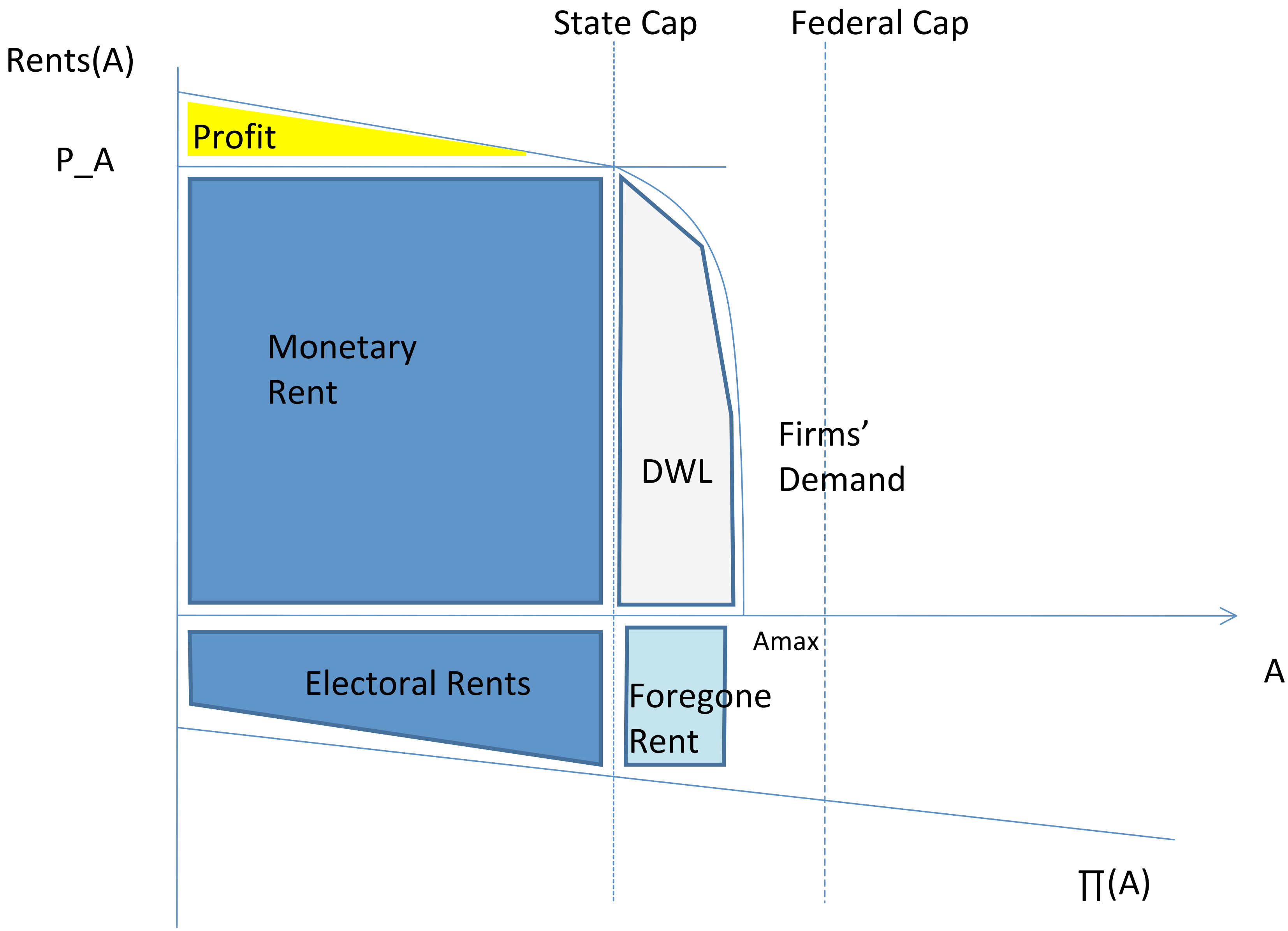


Rent(A)
Cap
Profit
P_A
Firms' Demand
Monetary Rents
A
Electoral Rents
∏(A)


Cap
Rent(A)
Profit
P_A
Net Rents
Firms'
Demand
∏(A)
A



Rent(A)
Cap
Firms'
Demand
Profit
P_A
Electoral Rents
A
∏(A)


State Cap
Federal Cap
Rents(A)
Profit
P_A
Monetary
Rent
DWL
Firms'
Demand
Amax
A
Electoral Rents
Foregone
Rent
∏(A)

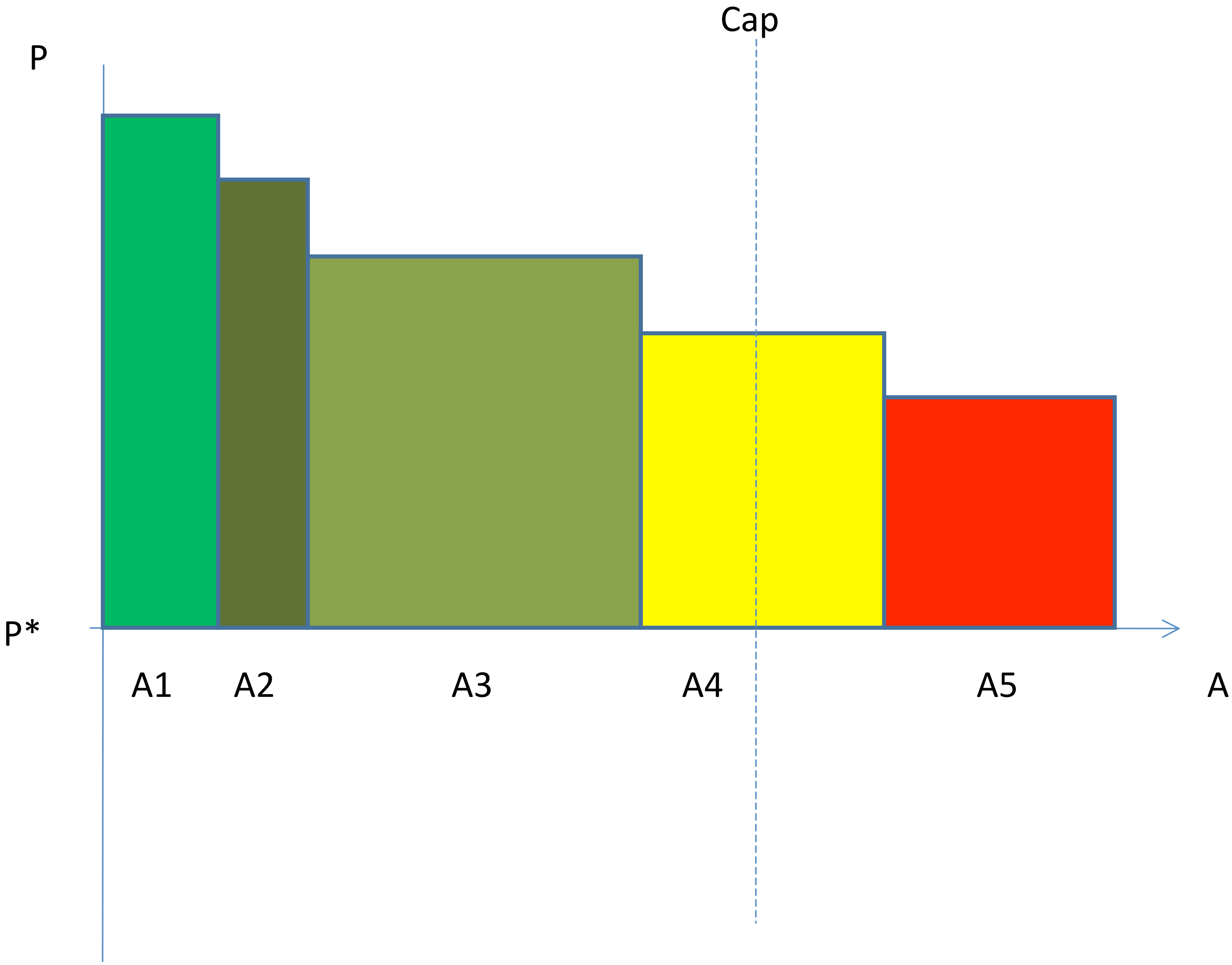
The Socially Optimal Allocation across Categories with Different Rents

V – Private Activity Volume Cap
D – measure of relevant GSP
F – coefficient relating relevant sector GSP to firm's demand
R – rank of category

$$\max_{\mathbf{A}} U(\mathbf{A}) = \mathbf{PA} \text{ s.t. } \mathbf{A_c} \leq \mathbf{F_c}(\mathbf{D_c}) \ \forall \ \mathbf{c} \text{ and } \sum_{\mathbf{C}} \mathbf{A_c} \leq \mathbf{V}$$

$$R_c = R(\mathbf{P_c})$$

$$A_c = \begin{cases} F_c D_c & \text{if } \sum_{R_j \leq R_c} F_j D_j < V \\ V - \sum_{j=1}^{R_c - 1} F_j D_j & \text{if } \sum_{R_j \leq R_c} F_j D_j > V \\ 0 & \text{if } \sum_{R_j < R_c} F_j D_j > V \end{cases}$$


Cap
P
P*
A1
A2
A3
A4
A5
A

The Politically Motivate Allocation across Categories with Different Rents - Assumptions

- All private activity tax-exempt borrowing involves for-profit firms
- Competitive markets set the profit from taxable bond borrowing to zero
- The public official knows
  - Potential rents (P) that could be created through bond financing
  - The firm's transfer rates (W)
  - The distribution of voter's preferences (H and G)
- Public officials announce a binding bond authority allocation schedule (**A**)
- The election is held, the authority is granted, bonds are issued, rents are realized and transferred

The Public Official's Expected Utility

- γ – baseline probability of election
- ψ – other rents of political office
- H – size of the constituency in the electorate
- G – political value of allocating to the constituency (benefit, preference and efficacy)
- W – constituency firms' transfer of rents

$$U(\mathbf{A}) = [\gamma + \sum_C G_c H_c A_c][\sum_C W_c P_c A_c + \psi]$$

$$\max_{\mathbf{A}} U(\mathbf{A}) \quad \text{s.t.} \quad \sum_C A_c \leq V \text{ and } A_c \leq F_c D_c \; \forall \; c$$

Solving for the Utility Maximizing **A**

$$
\begin{aligned}
L &= [\gamma + \sum_C G_c H_c A_c][\sum_C W_c P_c A_c + \psi] + \lambda_1 (\sum_C A_c - V) + \sum_C \lambda_{2c} (A_c - F_c D_c) \\
L &= [\gamma + G_c H_c A_c + \sum_{j \neq c} G_j H_j A_j][\sum_{j \neq c} W_j P_j A_j + W_c P_c A_c + \psi] - \lambda_1 (\sum_C A - V) - \sum_C \lambda_{2c} (A_c - F_c D_c)
\end{aligned}
$$

The first order conditions are:

$$
\begin{aligned}
\frac{\partial L}{\partial A_c} &= [G_c H_c][W_c P_c A_c + \sum_{j \neq c} W_j P_j A_j + \psi] + [W_c P_c][\gamma + G_c H_c A_c + \sum_{j \neq c} G_j H_j A_j] - \lambda_1 - \lambda_{2c} \leq 0 \ \forall \ c \\
\frac{\partial L}{\partial \lambda_1} &= \sum_C A_c - V \geq 0 \\
\frac{\partial L}{\partial \lambda_{2c}} &= A_c - F_c D_c \geq 0 \ \ \forall \ c \\
A_c \frac{\partial L}{\partial A_c} &= 0 \ \ \forall \ c \\
\lambda_1 \frac{\partial L}{\partial \lambda_1} &= 0 \\
\lambda_{2c} \frac{\partial L}{\partial \lambda_{2c}} &= 0 \ \ \forall \ c
\end{aligned}
$$

Solving for a form that can be estimated

$$A_c = \frac{[G_c H_c][\sum_{j\neq c} W_j P_j A_j + \psi] + [W_c P_c][\gamma + \sum_{j\neq c} G_j H_j A_j] - \lambda_1 - \lambda_{2c}}{-2G_c H_c W_c P_c}$$

$$A_c = \frac{\sum_{j\neq c} W_j P_j A_j + \psi}{-2W_c P_c} + \frac{\gamma + \sum_{j\neq c} G_j H_j A_j}{-2G_c H_c} + \frac{\lambda_1 + \lambda_{2c}}{2G_c H_c W_c P_c}$$

$$A_c = \frac{\psi}{-2W_c P_c} + \frac{\gamma}{-2G_c H_c} + \frac{\lambda_1 + \lambda_{2c}}{2G_c H_c W_c P_c} + \frac{\sum_{j\neq c} W_j P_j A_j}{-2W_c P_c} + \frac{\sum_{j\neq c} G_j H_j A_j}{-2G_c H_c}$$

$$A_c = \left(\frac{\psi}{W_c}\right)\frac{1}{-2P_c} + \left(\frac{\gamma}{G_c}\right)\frac{1}{-2H_c} + \left(\frac{\lambda_1 + \lambda_{2c}}{G_c W_c}\right)\frac{1}{2H_c P_c} + \sum_{j\neq c}\left[\left(\frac{W_j}{W_c}\right)\frac{P_j A_j}{-2P_c}\right] + \sum_{j\neq c}\left[\left(\frac{G_j}{G_c}\right)\frac{H_j A_j}{-2H_c}\right]$$

$$A_c = (\beta_1)\frac{1}{-2P_c} + (\beta_2)\frac{1}{-2H_c} + (\Phi)\frac{1}{2H_c P_c} + \sum_{j\neq c}\left[(\Theta_j)\frac{P_j A_j}{-2P_c}\right] + \sum_{j\neq c}\left[(\Psi_j)\frac{H_j A_j}{-2H_c}\right] + \epsilon$$

Data

- A, V – Bond Buyer Survey
- D – Bureau of Economic Analysis
  - Gross State Product by NAICS
- H – Current Population Survey
  - Percentage of households with a wage earner employed in the relevant industry
- P – Bloomberg, Mergent, Haver Analytics
  - The difference between the tax exempt bond coupons and taxable bond coupons for industrial development, utilities, and real estate
  - The difference between coupons and mortgage rates and a proxy for student loan interest rates

Descriptive Statistics

Variable	**Mean**	**Std. Dev.**	**Min**	**Max**
Volume Cap	$ 379.9	$ 365.1	$ 150.0	$ 2,384.6
Carry forward	$ 257.1	$ 315.4	$ -	$ 2,031.6
Total Cap	$ 700.2	$ 637.2	$ 150.0	$ 3,808.4
Total Borrowing	$ 338.4	$ 367.3	$ -	$ 2,476.6
Borrowing				
Industrial Development	$ 24.6	$ 37.4	$ -	$ 300.1
Utilities	$ 35.7	$ 71.2	$ -	$ 616.3
Mortgage Revenue Bonds	$ 106.8	$ 123.2	$ -	$ 778.5
Multifamily Housing	$ 90.7	$ 203.2	$ -	$ 1,417.7
Student Loans	$ 59.7	$ 83.4	$ -	$ 583.9
Other	$ 20.8	$ 65.9	$ -	$ 698.8

N=350 (2000-2006 for 50 states, excluding IL, including DC)
All figures are in millions of current year dollars.

Covariate Descriptive Statistics

Gross State Product	**Mean**	**Std. Dev.**	**Min**	**Max**
Industrial (Manufacturing)	$ 27,029.9	$ 30,698.1	$ 201.0	$ 182,992.0
Utilities	$ 4,286.5	$ 5,568.6	$ 305.0	$ 36,120.0
Mortgage Revenue (Real Estate)	$ 26,901.9	$ 39,170.4	$ 1,488.0	$ 293,958.0
Multifamily (Construction)	$ 9,979.7	$ 12,237.6	$ 691.0	$ 80,586.0
Student Loans (Education)	$ 1,902.1	$ 2,675.6	$ 30.0	$ 15,461.0
Interest Rate Advantage				
Industrial	2.50	2.46	-8.56	11.45
Utilities	1.69	1.68	-5.54	10.64
Mortgage Revenue (Mortgage Rates)	1.62	0.93	-0.59	6.54
Multifamily (Real Estate Bonds)	1.65	1.39	-2.92	6.64
Student Loans (LIBOR + .25)	-0.39	2.16	-9.96	4.92
Percentage of Households				
Industry (Manufacturing)	15.15	4.00	2.94	28.04
Utilities	1.26	0.49	0.35	3.26
Single Family (Real Estate)	1.96	0.69	0.24	4.35
Multifamily (Construction)	11.78	5.01	1.50	27.12
Student Loans (Education)	5.17	1.96	1.00	10.87

N=350 state-year observations. Dollar figures are in millions of current year dollars.

Reduced Form Regressions

	Industrial	Utilities	Mortgage	Multifamily	Student
Volume Cap/Pop	0.0167	-0.000161	0.410***	-0.0173	0.201**
	(0.019)	(0.019)	(0.085)	(0.015)	(0.087)
P Industrial	0.318	-0.124	-0.95	-1.691***	0.516
	(0.250)	(0.348)	(0.756)	(0.565)	(0.446)
P Utilities	0.222	0.299	0.71	0.335	0.106
	(0.247)	(0.398)	(1.379)	(0.495)	(0.831)
P Mortgage	-0.316	1.54	-3.361	0.207	1.989
	(0.859)	(1.033)	(5.303)	(1.125)	(1.645)
P Real Estate	0.316	0.693	-0.956	0.482	0.769
	(0.396)	(0.501)	(1.660)	(0.872)	(0.887)
P Student	0.445	0.356	-0.217	-0.366	-0.574
	(0.527)	(0.527)	(1.045)	(0.527)	(0.863)
H Industrial	0.36	-0.149	-1.061	0.222	0.174
	(0.360)	(0.479)	(0.782)	(0.331)	(0.792)
H Utility	1.842	10.19	-0.348	-3.932	-6.758
	(1.880)	(7.665)	(4.584)	(2.904)	(4.576)
H Real Estate	1.823	2.572	2.911	-0.863	2.081
	(1.710)	(2.444)	(4.378)	(3.259)	(3.025)
H Construction	-0.0805	0.23	-2.301***	0.663	0.258
	(0.176)	(0.295)	(0.878)	(0.537)	(0.755)
H Education	-1.266**	-1.117**	3.728**	-0.792	5.133**
	(0.642)	(0.533)	(1.751)	(0.798)	(2.282)
D Industrial/Pop	0.00109**	-0.000154	0.00424**	-0.00216**	-0.00226
	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)
D Utilities/Pop	0.0157	0.0245**	0.00112	-0.000824	-0.00276
	(0.012)	(0.012)	(0.018)	(0.004)	(0.014)
D Real Estate/Pop	-0.00205	0.0000117	-0.000421	-0.00109	-0.00791***
	(0.001)	(0.001)	(0.004)	(0.002)	(0.003)
D Construction/Pop	0.00462	0.00134	-0.0106	0.00557*	0.00547
	(0.004)	(0.003)	(0.008)	(0.003)	(0.006)
D Education/Pop	0.00722*	-0.00337	-0.0387***	0.0202**	0.00451
	(0.004)	(0.003)	(0.011)	(0.010)	(0.009)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Constant	-14.93	-30.34	30.02	16	7.932
	(19.030)	(32.830)	(30.310)	(15.800)	(17.450)
Observations	350	350	350	350	350
R-squared	0.199	0.206	0.282	0.195	0.149

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Structural Results

Tobit Models	Industrial	Utility	Mortgage	Multifamily	Student
psi/Wc	-16.20***	9.222	-37.12**	3.317	8.649
	(6.228)	(6.209)	(18.290)	(12.260)	(10.210)
gamma/Gc	-173.9*	95.93***	-121.8***	1457***	34.24
	(89.130)	(17.990)	(32.020)	(272.100)	(84.890)
Lambdas/GcWc	-291.3**	7.908	-68.12**	114.4**	35.25
	(118.200)	(5.892)	(31.520)	(52.250)	(37.390)
Widb/Wc		-0.0348**	0.118	0.0719	-0.00712
		(0.017)	(0.201)	(0.088)	(0.021)
Wutl/Wc	-0.0199		-0.0275	0.00729	-0.00514
	(0.022)		(0.063)	(0.022)	(0.015)
Wsfh/Wc	-0.00186	-0.0174**		-0.0121	-0.00196
	(0.015)	(0.009)		(0.057)	(0.016)
Wmfh/Wc	0.0061	0.0191**	-0.161***		0.0171
	(0.013)	(0.008)	(0.056)		(0.024)
Wstl/Wc	-0.0013	-0.00237	-0.0709	0.0032	
	(0.011)	(0.009)	(0.051)	(0.045)	
Gidb/Gc		-0.0563***	-0.0316	-0.0452	0.0575
		(0.017)	(0.035)	(0.234)	(0.063)
Gutl/Gc	-1.527*		-0.615***	-1.742	-1.731***
	(0.905)		(0.236)	(2.381)	(0.665)
Gsfh/Gc	-0.131	-0.141***		-8.647***	0.0719
	(0.301)	(0.041)		(0.741)	(0.277)
Gmfh/Gc	-0.0456	-0.0165***	-0.0940***		-0.0882**
	(0.041)	(0.004)	(0.019)		(0.039)
Gstl/Gc	-0.0793	-0.0606***	-0.0244	-1.874***	
	(0.151)	(0.022)	(0.052)	(0.549)	
Observations	350	350	350	350	350
R-squared	.	.	.	.	.

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

OLS w/correct SE	Industrial	Utility	Mortgage	Multifamily	Student
psi/Wc	-3.229	0.788	-23.57	-4.283	-0.385
	(2.427)	(0.838)	(18.560)	(5.224)	(1.899)
gamma/Gc	-390.2***	-22.69**	-143.5***	-233.2	-271.7***
	(103.700)	(9.432)	(36.240)	(202.300)	(74.010)
Lambdas/GcWc	-69.72*	0.465	-49.99	0.887	-0.755
	(39.160)	(0.343)	(38.280)	(12.540)	(4.745)
Widb/Wc		-0.00263	0.0603	0.00551	-0.00789
		(0.010)	(0.127)	(0.033)	(0.008)
Wutl/Wc	-0.0161***		-0.0722	0.00259	0.0000956
	(0.006)		(0.053)	(0.012)	(0.003)
Wsfh/Wc	0.00542	-0.00624***		0.00234	0.00232
	(0.004)	(0.002)		(0.032)	(0.003)
Wmfh/Wc	0.00797**	0.00891**	-0.0983**		-0.00395
	(0.003)	(0.004)	(0.039)		(0.004)
Wstl/Wc	0.0012	-0.00562	-0.0818**	-0.0107	
	(0.002)	(0.005)	(0.041)	(0.018)	
Gidb/Gc		-0.0126	-0.0433	-0.0219	0.0716
		(0.009)	(0.028)	(0.097)	(0.049)
Gutl/Gc	0.715		-0.246	-1.122	0.197
	(0.644)		(0.174)	(0.986)	(0.524)
Gsfh/Gc	-0.196	-0.0614		-1.477	0.207
	(0.139)	(0.046)		(0.958)	(0.160)
Gmfh/Gc	-0.00868	-0.0131***	-0.0437**		-0.0809***
	(0.028)	(0.005)	(0.022)		(0.030)
Gstl/Gc	0.0859	0.00204	-0.0271	-0.774**	
	(0.091)	(0.020)	(0.052)	(0.310)	
Observations	350	350	350	350	350
R-squared	0.064	0.167	0.198	0.085	0.081

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

$$A_c = \left(\frac{\psi}{W_c}\right)\frac{1}{-2P_c} + \left(\frac{\gamma}{G_c}\right)\frac{1}{-2H_c} + \left(\frac{\lambda_1+\lambda_{2c}}{G_cW_c}\right)\frac{1}{2H_cP_c} + \sum_{j\neq c}\left[\left(\frac{W_j}{W_c}\right)\frac{P_jA_j}{-2P_c}\right] + \sum_{j\neq c}\left[\left(\frac{G_j}{G_c}\right)\frac{H_jA_j}{-2H_c}\right]$$

$$A_c = (\beta_1)\frac{1}{-2P_c} + (\beta_2)\frac{1}{-2H_c} + (\Phi)\frac{1}{2H_cP_c} + \sum_{j\neq c}\left[(\Theta_j)\frac{P_jA_j}{-2P_c}\right] + \sum_{j\neq c}\left[(\Psi_j)\frac{H_jA_j}{-2H_c}\right] + \epsilon$$

Model Variation – Competitiveness

$$
\begin{aligned}
U(\mathbf{A}) &= [\gamma + \sum_C G_c H_c A_c][\sum_C W_c P_c A_c + \psi] \\
A_c &= \left(\frac{\psi}{W_c}\right)\frac{1}{-2P_c} + \left(\frac{1}{G_c}\right)\frac{\gamma}{-2H_c} + \left(\frac{\lambda_1 + \lambda_{2c}}{G_c W_c}\right)\frac{1}{2H_c P_c} + \sum_{j \neq c}\left[\left(\frac{W_j}{W_c}\right)\frac{P_j A_j}{-2P_c}\right] + \sum_{j \neq c}\left[\left(\frac{G_j}{G_c}\right)\frac{H_j A_j}{-2H_c}\right] \\
A_c &= (\beta_1)\frac{1}{-2P_c} + (\beta_2)\frac{\gamma}{-2H_c} + (\Phi)\frac{1}{2H_c P_c} + \sum_{j \neq c}\left[(\Theta_j)\frac{P_j A_j}{-2P_c}\right] + \sum_{j \neq c}\left[(\Psi_j)\frac{H_j A_j}{-2H_c}\right] + \epsilon
\end{aligned}
$$

Model Variation – Social Benefit

$$
\begin{aligned}
U(\mathbf{A}) &= [\gamma + \sum_C S_c B_c A_c + \sum_C G_c H_c A_c][\sum_C W_c P_c A_c + \psi] \\
A_c &= \frac{\sum_{j \neq c} W_j P_j A_j + \psi}{-2W_c P_c} + \frac{\gamma + \sum_{j \neq c}(S_j B_j + G_j H_j A_j}{-2G_c H_c} + \frac{\lambda_1 + \lambda_{2c}}{2(S_c B_c + G_c H_c) W_c P_c} \\
A_c &= \frac{\psi}{-2W_c P_c} + \frac{\gamma}{-2(S_c B_c + G_c H_c)} + \frac{\lambda_1 + \lambda_{2c}}{2(S_c B_c + G_c H_c) W_c P_c} + \frac{\sum_{j \neq c} W_j P_j A_j}{-2W_c P_c} + \frac{\sum_{j \neq c} G_j H_j A_j}{-2(S_c B_c + G_c H_c)}
\end{aligned}
$$

$$
\begin{aligned}
U(\mathbf{A}) &= [\gamma + \sum_C S_c B_c A_c][\sum_C W_c P_c A_c + \psi] \\
A_c &= \left(\frac{\psi}{W_c}\right) \frac{1}{-2P_c} + \left(\frac{\gamma}{S_c}\right) \frac{1}{-2B_c} + \left(\frac{\lambda_1 + \lambda_{2c}}{S_c W_c}\right) \frac{1}{2B_c P_c} + \sum_{j \neq c} \left[\left(\frac{W_j}{W_c}\right) \frac{P_j A_j}{-2P_c} \right] + \sum_{j \neq c} \left[\left(\frac{S_j}{S_c}\right) \frac{B_j A_j}{-2B_c} \right] \\
A_c &= (\beta_1) \frac{1}{-2P_c} + (\beta_2) \frac{1}{-2B_c} + (\Phi) \frac{1}{2B_c P_c} + \sum_{j \neq c} \left[(\Theta_j) \frac{P_j A_j}{-2P_c} \right] + \sum_{j \neq c} \left[(\Psi_j) \frac{B_j A_j}{-2B_c} \right] + \epsilon
\end{aligned}
$$

$$
\begin{aligned}
U(\mathbf{A}) &= [\gamma + \sum_C S_c A_c][\sum_C W_c P_c A_c + \psi] \\
A_c &= \left(\frac{\gamma}{S_c}\right) + \left(\frac{\psi}{W_c}\right) \frac{1}{-2P_c} + \left(\frac{\lambda_1 + \lambda_{2c}}{S_c W_c}\right) \frac{1}{2P_c} + \sum_{j \neq c} \left[\left(\frac{W_j}{W_c}\right) \frac{P_j A_j}{-2P_c} \right] + \sum_{j \neq c} \left[\left(\frac{S_j}{S_c}\right) \frac{A_j}{-2} \right] \\
A_c &= \alpha + (\beta_1) \frac{1}{-2P_c} + (\Phi) \frac{1}{2P_c} + \sum_{j \neq c} \left[(\Theta_j) \frac{P_j A_j}{-2P_c} \right] + \sum_{j \neq c} \left[(\Psi_j) \frac{A_j}{-2} \right] + \epsilon
\end{aligned}
$$

Model Variation – Organization

$$
\begin{aligned}
U(\mathbf{A}) &= [\gamma + \sum_C G_c O_c H_c A_c][\sum_C W_c P_c A_c + \psi] \\
A_c &= \left(\frac{\psi}{W_c}\right)\frac{1}{-2P_c} + \left(\frac{\gamma}{G_c}\right)\frac{1}{-2O_cH_c} + \left(\frac{\lambda_1 + \lambda_{2c}}{G_cW_c}\right)\frac{1}{2O_cH_cP_c} + \sum_{j\neq c}\left[\left(\frac{W_j}{W_c}\right)\frac{P_jA_j}{-2P_c}\right] + \sum_{j\neq c}\left[\left(\frac{G_j}{G_c}\right)\frac{O_jH_jA_j}{-2O_cH_c}\right] \\
A_c &= (\beta_1)\frac{1}{-2P_c} + (\beta_2)\frac{1}{-2O_cH_c} + (\Phi)\frac{1}{2O_cH_cP_c} + \sum_{j\neq c}\left[(\Theta_j)\frac{P_jA_j}{-2P_c}\right] + \sum_{j\neq c}\left[(\Psi_j)\frac{O_jH_jA_j}{-2O_cH_c}\right] + \epsilon
\end{aligned}
$$

$$
\begin{aligned}
U(\mathbf{A}) &= [\gamma + \sum_C G_c O_c A_c][\sum_C W_c P_c A_c + \psi] \\
A_c &= \left(\frac{\psi}{W_c}\right)\frac{1}{-2P_c} + \left(\frac{\gamma}{G_c}\right)\frac{1}{-2O_c} + \left(\frac{\lambda_1 + \lambda_{2c}}{G_cW_c}\right)\frac{1}{2O_cP_c} + \sum_{j\neq c}\left[\left(\frac{W_j}{W_c}\right)\frac{P_jA_j}{-2P_c}\right] + \sum_{j\neq c}\left[\left(\frac{G_j}{G_c}\right)\frac{O_jA_j}{-2O_c}\right] \\
A_c &= (\beta_1)\frac{1}{-2P_c} + (\beta_2)\frac{1}{-2O_c} + (\Phi)\frac{1}{2O_cP_c} + \sum_{j\neq c}\left[(\Theta_j)\frac{P_jA_j}{-2P_c}\right] + \sum_{j\neq c}\left[(\Psi_j)\frac{O_jA_j}{-2O_c}\right] + \epsilon
\end{aligned}
$$

Model Variation - Campaign Contributions

$$U(\mathbf{A}) \;=\; [\gamma + G\sum_{C} W_c A_c][\psi]$$

$$\frac{\partial U}{\partial A_c} = \psi G W_c$$

Recall the model:

$$\max_{\mathbf{A}} U(\mathbf{A}) = \mathbf{PA} \text{ s.t. } \mathbf{A_c} \leq \mathbf{F_c}(\mathbf{D_c}) \;\forall\; \mathbf{c} \text{ and } \sum_{\mathbf{C}} \mathbf{A_c} \leq \mathbf{V}$$

$$R_c = R(\mathbf{P_c})$$

Estimating F_c

$$A_c = \begin{cases} F_c D_c & \text{if } \sum_{R_j \le R_c} F_j D_j < V \\ V - \sum_{j=1}^{R_c - 1} F_j D_j & \text{if } \sum_{R_j \le R_c} F_j D_j > V \\ 0 & \text{if } \sum_{R_j < R_c} F_j D_j > V \end{cases}$$

$$E[A_c | A_c > 0] = P[M_c][V - \sum A_j] + [1 - P[M_c]][F_c D_c]$$

$$A_c = \beta_1 [V - \sum A_j] + \beta_2 D_c + \epsilon$$

$$F_c = \frac{[1 - P[M_c]] F_c}{1 - P[M_c]} = \frac{\beta_2}{1 - \beta_1}$$

Estimates of F_c

COEFFICIENT	Industrial	Utilities	Mortgage	Multifamily	Student
V-Sum(Aj)	-0.00323	-0.00384	0.142***	0.00936	0.168***
	(0.005)	(0.014)	(0.025)	(0.041)	(0.021)
GSP	0.000751***	0.00983***	0.00112***	0.0122***	0.00631**
	(0.000)	(0.002)	(0.000)	(0.002)	(0.003)
Observations	280	198	289	261	205
R-squared	0.402	0.656	0.651	0.751	0.717
Estimated F			0.0013	0.0123	0.0076

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

COEFFICIENT	Industrial	Utilities	Mortgage	Multifamily	Student
V-Sum(Aj)	-0.00624*	0.0541***	0.128***	0.277***	0.127***
	(0.003)	(0.019)	(0.022)	(0.045)	(0.020)
Ln GSP	3.530***	4.998***	7.040***	-1.357	7.704***
	(0.326)	(0.902)	(0.922)	(1.824)	(0.840)
Observations	280	198	289	261	205
R-squared	0.42	0.466	0.668	0.596	0.785
Estimated F		5.2839	8.0734	-1.8769	8.8247

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Questions and Suggestions